Title. Applications of discrete Morse theory to concurrency

Purpose of visit. Directed paths, which are a basic notion in directed topoogy, at first glance resemble gradient paths of a discrete vector field, which are a fundamental concept in discrete Morse theory. The aim of the visit was to look deeper into the connections between these two fields.

Bacground. The motivation for directed topology comes from concurrency theory. According to work by Martin Raussen, Lisbeth Fajstrup and others, the state space of (a specific type of) a concurrent program is a cubical complex X in $\mathbb{R}^n$, where n is the number of processors which use some common resources. The constraints are that each resource can be accessed only by a certain number of processes at the same time. The resources are represented as obstructions in the complex X in the form of points or, more generally, multirectangles. Raussen, Fajstrup and coauthors have shown that by decomposing X into finitely many pieces according to the how these obstruct excecution paths of the program, the trace space, that is the space of equivalence classes of excecution paths, decomposes into sub- trace spaces which are contractible and have contractible or empty intersections. Using this description, a topological model for the trace space is given by the nerve complex of this cover. According to recent work by Raussen and Meshulam, the trace space of a concurrent program can be, in most cases more efficiently, modelled as a configuration space.

Joint work. According to work by Raussen and Ziemiański, the trace space of a concurrent program on n processors is a subcomplex of a product of r spheres of dimension $(n-2)$, where r is the number of obstructions. A perfect discrete vector field on this ambinet space restricts naturally to a discrete vector field on the trace space with critical cells determined by the complement. This could give a more efficient way to compute the homology and provide additional insight into the structure of the trace space. We have tested the approach on some typical (small) examples, and it seems promising. The problem is that, although smaller than the original description of the trace space as a prodsimplicial complex, the nerve complex can be quite big even in relatively simple cases so that the computations are still quite complex. We have also attempted to find a Morse theoretic approach to the configuration space model, and looked into some other Morse theoretic approaches to directed paths, but this remained for the time being at the level of ideas which we hope to have the opportunity to look into in the future. We plan to continue the collaboration in the future, in this year at events scheduled within the ACAT framework (conference in Aalborg in April, summer school in Ljubljana in June, conference in Vienna in July), as well as in the future.

Conclusion. The visit was dedicated to studying the complex theory of directed algebraic topology which has been developed by Martin Raussen, Lisbeth Fajstrup and their coauthors, and to look for possible ways in which discrete Morse theory techniques could give additional insight into the relevant topological models. The idea of applying discrete Morse methods to this new domain seems promising, and we hope that it will lead to new results. I would like to sincerely thank my hosts Lisbeth Fajstrup and Martin Raussen for the effort that they have put into explaining to me the relevant concepts from directed algebraic topology and to the project of bringing discrete Morse thoery into it. I would also like to thank the Department of Mathematics at Aalborg University for their hospitality and the pleasant time that I spent there (and in particular Ama for arranging for the bike.)