Research Networking Programmes

Short Visit Grant or Exchange Visit Grant

(please tick the relevant box)

Scientific Report

The scientific report (WORD or PDF file – maximum of eight A4 pages) should be submitted online within one month of the event. It will be published on the ESF website.

Proposal Title: *Invariants of knots and knotted graphs from point clouds*

Application Reference N°: *7119*

1) Purpose of the visit

Initiate collaboration on biologically motivated problems from computational geometry**.**

2) Description of the work carried out during the visit

A talk at the IST geometry and topology seminar on 11th March 2015.
Title: *a homologically persistent skeleton in computer vision and beyond*.
The preprint is available at http://kurlin.org/projects/hopes.pdf (26 pages, 3.1M).

Prof Herbert Edelsbrunner has suggested the problem to find an optimal packing of balls with limited overlap. More exactly, we consider the arrangement of equal balls with centers at all nodes of a lattice in a Euclidean space.

The classical problem is to find a lattice and a radius such that (1) all balls don't overlap, (2) the probability that a random point is covered by **at least one** ball is maximal.

Our problem with optimal overlap is to find a lattice and a radius of balls such that (2') the probability that a random point is covered by **exactly one** ball is maximal.

The motivation comes from practical arrangement of tightly packed chromosomes in a cell nucleus. Each chromosome looks like a bit squashed ball. However there should be enough room to allow chromosomes to move in and out the cell nucleus for future cell divisions. Hence we model all chromosomes by equal balls that partially overlap.

3) Description of the main results obtained

We have student the 2-dimensional case in details and have almost finished a proof that a hexagonal lattice provides an optimal packing with limited overlap. Any 2-dimensional lattice can be described by 2 parameters (up to scaling). An optimal packing was known only for a 1-parameter family of so-called delta-distorted lattices. Our proof covers the general 2-dimensional case, which turned out to be rather computationally complicated. It remains to finish numerical simulations to confirm that the maximal probability of about 93% is achieved by a hexagonal lattice with size 1 and a radius of balls about 0.52.

4) Future collaboration with host institution (if applicable)

Prof Herbert Edelsbrunner has invited me to the final ACAT meeting at IST on 6-10 July 2015 to meet Prof Yasuaki Haraoka (Japan), who is also working on optimal packings and persistent homology. We plan to extend our results about optimal packings of balls with optimal overlap from dimension 2 to 3.

5) **Projected publications / articles resulting or to result from the grant *(ESF must be acknowledged in publications resulting from the grantee's work in relation with the grant)***

A joint preprint with Prof Herbert Edelsbrunner and his PhD student Mabel Iglesias-Ham is in preparation. The support of the ACAT network will be acknowledged.

6) Other comments (if any)