Scientific report - Grzegorz Jabłoński
Visit in Bologna (Host: dr Patrizio Frosini).

1. Purpose of the visit:
   Work on G-invariant persistent homology with focus on computational aspects. Our aim was based on the paper "G-invariant persistent homology" by Patrizio Frosini, which is the foundation of the new approach to the persitent homology. We would like to devise new algorithms and applications.

2. Description of the work carried out during the visit:
   Classical persistent homology is well tested and used tool, however we would like to point out it is not able to study the action of transformation groups that are different from the group of all self-homeomorphisms of a topological space. The main aim of our research is to provide theory that will be useful in study of spaces with respect to some transformation (for example rigid motions). Our work during my visit was focused on following problems:
   1. general ideas from the paper "G-invariant persistent homology". We prepared some examples where classical persistent homology is not sufficient, and our methods might work better. This examples include filtration on circle S^1 and rotation of images of homeomorphic letters (P and Q, I and L)
   2. representation of G-invarian persistent homology of some topological space X with some other group H. We could provide suitable conditions on group H, such that chains constructed with the group H will be G-invariant. Then we can take the quotient space X/H of the topological space X. Computing persistent homology of that space is not a hard theoretical or computational problem. However it is not obvious how to choose proper group H.
   3. how to compute G-invariant persistent homology when group H is simple and G is group of rigid motions. We decided to choose circe S^1 as our topological space. Then we define filtration on a complex isomorphic to S^1. As H we can choose the group consisting of two functions: the identity map and the antipodal map on S^1. As admissible chains we take chains that are invariant under antipodal map. It is then easy to compute the X/H – we just identify simplices that are antipodal ("glue" them together). Then we can use classical persistent homology algorithm

3. Description of the main results obtained:
   We devised some applications of G-invariant persistent homology – these are objects that are homeomorphic but not invariant with respect to rotations (letters case). We also proved that the persistent homology of quotient space X/H for suitable choice of H for the given G is an example of G-invariant persisten homology. This result may looks simple, but we stated the hypothesis that this is not a case for all spaces X and groups G.
   Finally we prepared modifications to the classical persistent homology algorithm that allow us to compute persistent homology groups of the space X/H. We think this direction of research will be very fruitful and might be applicable to different problems in shape recognition and image comparision.

4. Future collaboration with host institution
   We plan future collaboration with dr Patrizio Frossini, because the topic is interesting and potential applications were defined. We also prepared workplan for the next months:
   1. prepare the implementation of the software to compute X/H where X is circle and H is group of rotations preserving polygons.
   2. decide how one can choose groups H if the group G is given

5. Projected publications / articles resulting or to result from the grant
   We plan to write short paper about results obtained during my visit. We hope our article will have response from computational topology researchers.