Scientific Report

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Short visit to Professor Graham Ellis, National University of Ireland - Galway

Contents

1 Purpose of the visit

The purpose of the visit was to discuss the general idea of our work and to set the right scenario in which to implement the theoretical constructions I am building for my PhD thesis. Specifically, which algebraic objects and programming environment to use when dealing with what we call A_∞-persistence (work still in progress).

Under the direction of Professor Aniceto Murillo in the University of Malaga (Spain), I am currently working on finding a compatible theory of Persistence for invariants more powerful than homology, in order to obtain more information than what we can achieve through Persistent Homology.

Persistent Homology has applications in a wide range of areas as Statistics combined with Biology, Medicine or Text Mining, it can be used to deal with high dimensional data sets, to analyse images, etc. As the notion of Persistent Homology spreads through the scientific community (specially in branches other than mathematics), we can expect to find more and more applications. That is why our research can be useful not only at a theoretical level, for if we get to interpret the extra information we are looking for, we

will be able to contribute in lots of areas. With this in mind, we keep an algorithmic perspective, towards implementing what we develop.

On the other hand, the computational part of this theory is not only interesting because of its applications. The fact that it strongly uses techniques from Discrete Morse Theory and the point of view of barcodes/matrices makes it an exciting ground to work in.

Since Professor Graham Ellis at the National University of Ireland Galway has worked for many years with algebraic - topological tools and computers (programming some on his own [HAP]), I decided to pay him a visit (May 2-16, 2013).

2 Description of the work carried out during the visit

After showing each other some of our latest ideas, Professor Ellis recommended me some papers I could be interested in, such as [C04], [DKM10], [KM12], [V-J08].

We discussed algorithms, approaches to my constructions, programming environments, a kind of barcodes I try to build for the persistent information I operate with, etc. We did not do it only in private meetings, but also by participating in an informal seminar. Bui Anh Tuan, Le Van Luyen (PhD candidates) and Alexander D. Rham (PostDoc), three students Graham Ellis supervises, and I gave talks about our work.

Not long before the short visit, I started implementing algorithms to apply on filtrations of simplicial complexes what my advisor Aniceto Murillo and I are currently developing. This was a first step to check everything could work out fine.

However, in my work, I use Alexander-Whitney diagonal for simplicial complexes, and this must be arranged if one wishes to work with other structures like cubical sets, etc. After some of Professor Ellis' remarks, I decided it would be good for generality and applicability reasons to work with cubical sets instead of simplicial complexes, and he gave me references where I could find tools I would need for that change, like a diagonal approximation for cubical sets [DKM10], [KM12]. With this in mind, he also introduced me to GAP [GAP], a freely distributed system for computational discrete algebra, with particular emphasis on Computational Group Theory. GAP provides a programming language and functions and data libraries to work with algebraic objects. Such software is specially suitable to carry out that change I am aiming to do, for it allows to get suitable input (e.g. transforming images into filtered pure cubical complex parametrized by a threshold [HAP]), to work algebraically and plot some results, all in one, with a language similar to that of C.

3 Description of the main results obtained

As we said, the purpose of the visit was to discuss the general idea of our work and to choose suitable algebraic objects and programming environment to go on with the theory, so that we can use the software to get examples supporting the interest of our work and to apply it to real world problems.

We reached that goal by deciding to move from programming in C++ with (filtrations of) simplicial complexes (the setting of our first tests) to doing it in GAP with (those of) cubical sets. And we chose that because of the following reasons.

Images are usually discretized by cubes (voxels, the 3D generalizations of pixels), or more generally, by cubical complexes. There is also software available to easily produce 3D images by cubes (volume sculpting) [Vox]. Since one of our main purposes is to apply our theory to Image Analysis, like the authors do in [RSW11] with Persistent Homology, it makes sense to use that kind of topological-algebraic objects, from the applications viewpoint.

This also suits from a technical point of view, given that we have the tools we need for cubical complexes (cubical homology) [KMM04], [DKM10], [KM12] in order to build our new constructions expressing the A_∞-persistence.

4 Future collaboration with host institution (if applicable)

Two points might lead to future collaboration.

On the one hand, this group in NUI Galway has more experience with computations on the cohomology ring of a space than on A_∞-coalgebra structures on $H_*(X)$. Our algorithmic approach focuses right now on the latter, while at some point it could be interesting to sort of dualize to cohomology, where there may be a broader intersection between our research group's interests.

On the other hand, although NUI Galway students' topics are more algebraic and mine is more topological (their point of view studies groups per se while I only deal with groups as an algebraic structure associated to a space), we share the use of tools like Perturbation Lemmas [C04] (although not the exact same version).

5 Projected publications / articles resulting or to result from the grant (ESF must be acknowledged in publications resulting from the grantee's work in relation with the grant)

When we implement -for cubical complexes with GAP- our (still in progress) work on A_∞-persistence, once Aniceto Murillo and I publish such work, the ESF will be properly acknowledge.

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