August 21, 2013

Paweł Pilarczyk

Algorithms for the computation of cohomology operations in the context of cubical complexes

The purpose of this short visit was to work with Dr. Pedro Real on an algorithmic approach to the computation of cohomological operations in the context of cubical complexes.

During this visit, a computational framework was designed and implemented for the construction of a simplicial set that represents a cubical set by means of the Cartesian product of the simplicial sets that correspond to the edges of the cubical cells. This simplicial set is endowed with appropriate face and degeneracy operators, as required by the theory [see e.g. J. Peter May, Simplicial objects in algebraic topology, The University of Chicago Press, Chicago and London, 1967]. These operators are inherited from the corresponding operators on the simplicial sets that represent the edges of the cubical cells. In order to achieve this goal, a container class was designed and implemented, which represents a simplicial set and has some useful additional operations (e.g. adding all the faces, to make it a valid complex, or normalizing, that is, removing degenerate cells). Moreover, a class that represents a product of simplicial cells was designed and implemented, with the ability to represent products of products in a recursive way. In addition to that, a previously prepared class that represents a simplex was updated so that the degeneracy operators are as easy to use as face operators.

For testing purposes, this framework was plugged into a previously programmed procedure for the computation of homology groups, cohomology groups, and the cup product in cohomology, and -- as expected -- the computations conducted on the simplicial set constructed from a collection of cubical cells provide the same results as the computations that use cubical cells directly.

Within this framework, the formulas for the Alexander-Whitney operator AW, the Eilenberg-Mac Lane operator EML, and the Shih operator SHI have been implemented, using the theoretical basis contained in the paper by R. González-Díaz and P. Real [Journal of Pure and Applied Algebra 139 (1999), 89-108]; see also [P. Real, Homological perturbation theory and associativity; Homology, Homotopy and Applications 2 (2000), 51-88; especially pg. 56 for the formulas]. This constitutes the Eilenberg-Zilber chain contraction, which provides a homological equivalence of the chain complex of the Cartesian product of edges of a cubical cell to the tensor product of chain complexes of the edges (the latter isomorphic with the cubical complex of the cubical cell).

The new data structures and procedures are contained in an updated version of the C++ source code package *chaincon*, which was uploaded to the website [http://www.pawelpilarczyk.com/chaincon/]. In addition to the new data structures and procedures, a program *cubsimprod.cpp* was added to demonstrate this machinery.

The designed and implemented computational algebraic-topological framework constitutes a basic tool for transferring cohomological operations from simplicial to cubical context, which will be a subject for further academic collaboration between Dr. Paweł Pilarczyk and Dr. Pedro Real. This framework opens immense possibilities for interesting applications. It is expected that the results of the work conducted by this team, possibly also involving some students, will provide grounds for a few papers on combinatorial procedures for the computation of cohomological operations in the context of cubical complexes, which will be published in international peer-refereed journals in the near future.

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Travel Costs Explanation

I was traveling from Porto to Seville and back by car (about 600 km each way). The cheapest airplane tickets which I found at the time I was applying for the ESF grant cost 312 euro (I am attaching a screenshot from Iberia airlines website below). Therefore, following the rules for travel cost reimbursement, I would like to kindly ask for reimbursement of this amount.