CAST TRAVEL GRANT REPORT
ENERGY-CAPACITY INEQUALITIES FOR THE COISOTROPIC HOFER-ZEHNDER CAPACITY

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The following is the scientific report for the CAST short visit grant for a 10 day visit to the University of Nantes from February 20th - 28th, and March 13th-15th, 2014 to work with Samuel Lisi.

1. Purpose of the visit

The purpose of this visit was to continue work on a Hofer-Zehnder type capacity $c(M,N)$ defined relative to co-isotropic submanifolds that we had developed earlier. We had established an energy-capacity inequality of the form $c(M, N \cap M) \leq e(N, M)$, where N here is a Lagrangian submanifold such that the Lagrangian spectral invariants of Leclercq [3] are well-defined, i.e. $\mu|_{\pi_2(M,L)} = 0$ and $\omega|_{\pi_2(M,L)} = 0$, and $e(N,M)$ is the minimal energy required to displace N from M. Our hope was to show that a version of this inequality is true for other Lagrangian and coisotropic submanifolds, and prepare several of our earlier results for publication.

2. Work carried out during the visit, main results obtained

In earlier work [4], we had constructed a Hofer-Zehnder type capacity $c(M,N)$ for pairs (M,N) where $N \hookrightarrow M$ is a properly embedded coisotropic submanifold. The definition is analogous to the definition of the Hofer-Zehnder capacity [2]. We then showed that $c(B(1), B_N(1)) = c(Z(1), Z_N(1)) < \infty$, where $B_N(1)$ and $Z_N(1)$ are the $n+k$-dimensional 'coisotropic parts' of $B(1)$ and $R^{2n-2} \times B^2(1)$, respectively, where the balls are considered to be centered at the origin in $\mathbb{C}^n$. In addition, we have established one important case of an energy-capacity inequality for this capacity.

Our main effort during this visit was spent considering several new settings in which we believe that an energy-capacity inequality holds. The results we were looking for have the following form:

Conjecture 1. *Let M^{2n} be a symplectic manifold, $U \subset M$ be a $2n$-dimensional symplectic submanifold, and $N \hookrightarrow M$ an $(n+k)$-dimensional coisotropic submanifold of M with $N \cap U \neq \emptyset$. Then $c(U, N \cap U) \leq \alpha e(N, U)$ for some $\alpha > 0$.*

This conjecture is known not to hold in general due to a result of Rizell [6], and so it is very interesting to know when for what classes of coisotropic submanifolds it holds. During this visit, we first detailed a variational proof showing that the energy capacity holds when M is an open subset of $\mathbb{R}^{2n}$ and N is the intersection of M with an $(n+k)$-dimensional coisotropic affine space. Our proof follows closely that of Hofer-Zehnder's variational proof for the energy-capacity inequality between the Hofer-Zehnder capacity and the displacement energy of an open subset of $\mathbb{R}^{2n}$ [2], but also adapts several ideas from Usher [8] and Oh [5] to our setting.

We then directed our attention to Lagrangian versions of symplectic homology. We were able to show that, when (U, N) is a pair such that N is Lagrangian and the Lagrangian version of symplectic homology Viterbo [9] or Floer, Hofer, and Wysocki [1] are well defined, then the respective capacities coming from the symplectic homology theories are upper bounds for our Hofer-Zehnder-like capacity. The proof also gives the analogous result in the absolute (i.e. purely symplectic) case for the Hofer-Zehnder capacity. While the general relationship between the Viterbo and Floer, Hofer, Wysocki versions of symplectic homology is unknown, our methods of proof for the above lead us to the conjecture that at least the capacities defined from them are equal.

3. Future collaboration with the host institution

Future collaboration with the host institution is extremely likely. We would still like to construct spectral invariants and establish an energy-capacity inequality for displaceable monotone Lagrangians and other classes of coisotropic submanifolds. We are also interested in the possibility that our coisotropic energy-capacity results can be used to establish the non-degeneracy of the Chekanov-Hofer metric (see, for instance, [7] for a definition) for certain classes of coisotropic submanifolds. We would additionally like to continue work on our conjecture that the symplectic homology capacities from both the Viterbo and Floer, Hofer, Wysocki versions of symplectic homology are equal.

4. Projected publications resulting from this grant

The results obtained during the period of the grant will form approximately the second half of a paper already in progress, as well as reinforce a section on applications in a second paper.

References

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