ÉVELINE LEGENDRE'S VISIT TO LISBON, MAY 28TH TO JUNE 6TH

1. Introduction

I visited Rosa Sena-Dias in Instituto Superior Técnico, Lisbon from May 28th to June 6th. During this short visit we worked on some ongoing projects and were able to make some progress.

2. Upper on invariant λ_1 on toric manifolds

As pointed out by Polterovich (Crelle's, 1998), there is a upper bound on the first eigenvalue of Kähler metrics having a rational Kähler class. The question of wether there is such a bound for irrational Kähler classes is still open. During my visit, we understood how to obtain an upper bound for the first eigenvalue of toric kähler metrics on toric manifolds whose Kähler class is not rational. This follows from the fact that all toric manifolds are projective. In fact one can explicitly write down an estimate for this bound in terms of moment polytope data. It is possible that in the rational cases our bound is better than that of Polterovich.

3. Lower on invariant λ_1 on toric manifolds

Abreu-Freitas construct a family of toric Kähler metrics on S^2 with unbounded invariant λ_1. We have a potential extension of their method to all toric manifolds and construct several families of metrics which potentially will have arbitrarily small eigenvalue. We have reduced the unboundedness question for these families to a Hardy type inequality on the moment polytope which we plan to study subsequently.

4. Generic spectrum of Kähler manifolds

We are also interested in the question of weather generic Kähler metrics have simple spectrum. Together with V. Guillemin we have a method to attack this question and we reduced it to a question about the kernel of the operator $\bar{\partial}^*\partial^*$. We looked at this kernel in some special cases (for example on $\mathbb{C}P^n$) to see if we can extract enough information to settle the question there.