CAST TRAVEL GRANT REPORT
LAGRANGIAN FLOER THEORY FOR LAGRANGIAN COBORDISMS BETWEEN LEGENDRIAN SUBMANIFOLDS

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The following is the scientific report for the CAST short visit grant for a 4 days visit to Pisa, Italy for the period November 21st - 24th to work with Baptiste Chantraine, Georgios Dimitroglou Rizell and Paolo Ghiggini.

1) Purpose of the visit. The goal of my trip to Pisa, Italy was to meet with Baptiste Chantraine (University of Nantes), Georgios Dimitroglou Rizell (Cambridge University) and Paolo Ghiggini (Nantes – but currently long term visitor in Pisa) and to work on our projects on Floer theory for exact Lagrangian cobordisms between Legendrian submanifolds.

2) Description of the work carried out during the visit. Baptiste Chantraine, Georgios Dimitroglou Rizell, Paolo Ghiggini and I are developing a Floer theory for a pair of exact Lagrangian cobordisms inside an exact symplectic cobordism. Lagrangian intersection Floer homology was invented by Floer, who considered the closed case. In the case of Lagrangian cobordisms with non-compact cylindrical ends inside the convex end of a symplectic cobordism, a variant of Lagrangian intersection Floer homology, called wrapped Floer homology, was constructed by Abbondandolo–Schwarz, Abouzaid and Ekholm. The homology theory that we develop is a further generalization of wrapped Floer homology to the case when the Lagrangian cobordisms, in addition to the above non-compactness, are allowed to also have non-compact cylindrical ends in the concave part of the symplectic cobordism. In order to define the theory, we are required to impose the condition that the ends are cylindrical over Legendrians whose Chekanov-Eliashberg algebras admit augmentations (this should be seen as a requirement of non-obstructedness).

3) Description of the main results obtained. The main applications of this Floer theory that we get are, first, several long exact sequences relating the linearised Legendrian contact homologies of the Legendrian submanifolds being the ends of an exact Lagrangian cobordism.

Theorem 1. *Let L be an exact graded Lagrangian cobordism from Λ_- to Λ_+, and let ε_0^- and ε_1^- be two augmentations of Λ_- inducing augmentations ε_0^+, ε_1^+ of Λ_+, then there are the following three long exact sequences:*

$$\cdots \to LCH^{k-1}_{\varepsilon_0^+,\varepsilon_1^+}(\Lambda_+) \to H_{n+1-k}(L,\Lambda_-) \to LCH^{k}_{\varepsilon_0^-,\varepsilon_1^-}(\Lambda_-) \to LCH^{k}_{\varepsilon_0^+,\varepsilon_1^+}(\Lambda_+) \to \dots,$$

$$\cdots \to LCH^{k}_{\varepsilon_0^+,\varepsilon_1^+}(\Lambda_+) \to LCH^{\varepsilon_0^-,\varepsilon_1^-}_{n-k-1}(\Lambda_-) \to H^{n-k-1}(L) \to LCH^{k+1}_{\varepsilon_0^+,\varepsilon_1^+}(\Lambda_+) \to \dots,$$

$$\cdots \to LCH^{k-1}_{\varepsilon_0^+,\varepsilon_1^+}(\Lambda_+) \to H_{n-k}(\Lambda_-) \to LCH^{k}_{\varepsilon_0^-,\varepsilon_1^-}(\Lambda_-) \oplus H_{n-k}(L) \to LCH^{k}_{\varepsilon_0^+,\varepsilon_1^+}(\Lambda_+) \to \dots.$$

Using these long exact sequences, we are able to obtain strong restrictions on the topology of an exact Lagrangian cobordism in terms of its ends.

Theorem 2. *Let Λ be a Legendrian homology sphere in $P \times \mathbb{R}$, and let L be an exact Lagrangian endocobordism of Λ. If $\mathcal{A}(\Lambda)$ admits an augmentation, then L is a homology cylinder, i.e. $H_*(L, \Lambda) = 0$.*

Theorem 3. *Let L be an exact Lagrangian cobordism from the Whitney Legendrian sphere to itself, then L is an h-cobordism.*

4) Future collaboration with host institution. Currently I do not have any common projects with host institution.

5) Projected publications. The work on Floer theory for Lagrangian cobordisms is in preparation and will be posted on arxiv in the nearest future.