Scientific reports

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1. Purpose of the visit

   This workshop is unique in the sense that almost all top-level researchers in set theory gather from all over the world. In this visit, I would like to discuss with them about the relevant questions for my research as well as many topics in set theory and broaden my interest in set theory.

2. Description of the work carried out during the visit

   I gave a talk about Blackwell determinacy on June 16th and Hugh Woodin gave me an interesting comment about the topic and we discussed after the talk. I also discussed with Grigor Sargsyan about the related topic. After the talk by Benedikt Löwe, I found some generalization of the results given in his talk.

   Besides that, I listened to many interesting talks in large cardinals and descriptive set theory.

3. Description of the main results obtained

   (a) On the axiom of real Blackwell determinacy.

   Hugh Woodin observed that the axiom of real Blackwell determinacy ($\text{Bl-AD}_{\mathbb{R}}$) implies that the determinacy of all sets of reals in $\mathrm{L}(\mathbb{R}, \mathbb{R}^{n\#})$ for any natural number n, where $\mathbb{R}^{n\#}$ is the n-th iterate of the sharp operation starting from $\mathbb{R}$.

   With Grigor Sargsyan, we have observed that the determinacy of all sets of reals in $\mathrm{L}(\mathbb{R}, \mathbb{R}^{n\#})$ is equiconsistent with infinitary many Woodin cardinals with ${V_\delta}^{n\#}$, where δ is the supremum of the Woodin cardinals, for any natural number n. Also we have observed that the consistency strength of the above statements are much stronger than that of $\mathrm{AD}^{\mathrm{L}(\mathbb{R})}$ with the existence of $\mathbb{R}^{n\#}$.

Combining these two results, we can conclude that the consistency of Bl-$\mathrm{AD}_{\mathbb{R}}$ is strictly stronger than that of infinitary many Woodin cardinals with ${V_\delta}^{n\#}$ for each n. This is new and stronger than what we had before the workshop.

We believe that Bl-$\mathrm{AD}_{\mathbb{R}}$ implies that the determinacy of sets of reals in $\mathrm{L}(\mathbb{R}, \mu)$, where μ is the supercompact measure on the set of all countable subsets of the reals, which is much stronger than what we have so far but we have not proved it yet.

(b) On the implications between regularity properties.

I have found that if every $\mathbf{\Sigma}^1_3$-set of reals is Lebesgue measurable, then every $\mathbf{\Sigma}^1_3$-set of reals has the property of Baire assuming sharps for reals. From this, if one would like to force the statement that every $\mathbf{\Sigma}^1_3$-set of reals has the property of Baire but there is a $\mathbf{\Sigma}^1_3$ non-Lebesgue measurable set, then one must either start from the universe without sharps for some sets, or one has to use class forcings.

It is most likely that all the known implications and non-implications between typical regularity properties for $\mathbf{\Sigma}^1_2$-sets of reals and $\mathbf{\Delta}^1_2$-sets of reals can be generalized to the ones between typical regularity properties for $\mathbf{\Sigma}^1_3$-sets of reals and $\mathbf{\Delta}^1_3$-sets of reals assuming sharps for sets. But we have not checked it yet.

4. Projected publications/articles resulting or to result from the grant.

The results about Blackwell determinacy obtained in this workshop will be in my Ph. D thesis unless there are more efficient improvements.