NEW FRONTIERS OF INFINITY
SHORT VISIT GRANT - FINAL REPORT

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• Purpose of the visit

I have been invited by Professor Boban Velickovic of the university of Paris 7, to attend the *Logic Colloquium 2010* conference (July 2010, at the university of Paris 7), give an invited special session talk on the *Generalized Souslin Hypothesis*, and discuss this problem with the attendants of the conference.

• Description of the work carried out during the visit

The contrapositive formulation of the *Generalized Souslin Hypothesis* asserts that the Generalized Continuum Hypothesis together with $\square^*_\lambda$ implies the existence of a λ^+-Souslin tree for every uncountable cardinal λ. I have been looking at all known results on this problem, and seeked for a formal way of unifying all of them. A preliminary list of results has been presented at the *Set Theory* special session of the *Logic Colloquium 2010.* Consequently, a considerable list of inputs have been provided on this work, and a couple of open questions has been resolved. See below.

• Description of the main results obtained

Previously, I have isolated a combinatorial principle, denoted $\boxed{\diamondsuit}^\Gamma_{\lambda,<\mu}$, and established that it implies the existence of a λ^+-Souslin tree, and that the classical results concerning the existence of a λ^+-Souslin tree can go through this proxy principle. This includes Jensen's theorem that $\lambda^{<\lambda} = \lambda + \diamondsuit_{E^{\lambda^+}_\lambda}$ implies the existence of such tree, Jensen's theorem that $\mathsf{GCH} + \square_\lambda$ implies the existence of such tree, Gregory's theorem that $\lambda^{<\lambda} = \lambda + \mathrm{CH}_\lambda$ together with the existence of a non-reflecting stationary subset of $E^{\lambda^+}_{<\lambda}$ implies the existence of such tree, and Shelah's theorem that CH together with the saturation of the non-stationary ideal on ω_1, implies the existence of an ω_1-Souslin tree.

During my stay, I have extended the above work to cover also the 2007 result of König, Larson and Yoshinobu who proved that $⅄^*(\kappa, E^{\lambda^+}_\lambda)$ implies the existence of a λ^+-Souslin tree for all $\kappa \leq \lambda$. I was also able to prove an analogue of Gregory's theorem for the case that λ is a singular cardinal.

Altogether, we now get a very clear, coherent and comprehensive picture on the interplay between generalized Souslin trees and the above-mentioned combinatorial principles. Specifically, we get that:

Theorem 0.1. *Let λ denote an uncountable cardinal. Then:*

(1) *if $\lambda^{<\lambda} = \lambda$ and $\diamondsuit(E^{\lambda^+}_\lambda)$ holds, then $\boxed{\diamondsuit}^\Gamma_{\lambda,<\lambda^+}$ is valid;*
(2) *if $\lambda^{<\lambda} = \lambda$ and $⅄^-(\kappa, E^{\lambda^+}_\lambda)$ holds for some $\kappa \leq \lambda$, then $\boxed{\diamondsuit}^\Gamma_{\lambda,<\lambda^+}$ is valid;*
(3) *$\square_\lambda + \mathrm{CH}_\lambda$ is **equivalent** to $\boxed{\diamondsuit}^\Gamma_{\lambda,1}$;*
(4) *$\square_{\lambda,<\mu} + \mathrm{CH}_\lambda$ is **equivalent** to $\boxed{\diamondsuit}^\Gamma_{\lambda,<\mu}$ for all $\mu \leq \mathrm{cf}(\lambda)$;*
(5) *if $2^{<\lambda} = \lambda$ and there exists a non-reflecting stationary subset of $E^{\lambda^+}_{\neq\mathrm{cf}(\lambda)}$, then $\square^*_\lambda + \mathrm{CH}_\lambda$ is **equivalent** to $\boxed{\diamondsuit}^\Gamma_{\lambda,<\lambda^+}$. In particular:*

(6) *if* $\lambda^{<\lambda} = \lambda$ *and there exists a non-reflecting stationary subset of* $E^{\lambda^+}_{<\mathrm{cf}(\lambda)}$*, then* $\diamondsuit^{\Gamma}_{\lambda,<\lambda^+}$ *is valid.*

The above captures, and also extends, **all** known sufficient conditions for the existence of a λ^+-Souslin tree.

Theorem 0.2. *Let* λ *denote an uncountable cardinal. Then:*

(1) $\diamondsuit^{\Gamma}_{\lambda,<\mu}$ *implies the existence of a* λ^+*-Souslin tree, for every* $\mu \leq \lambda^+$*. Moreover, the tree may be rigid, and if* $\lambda^{<\kappa} = \lambda$ *and* $\Gamma \setminus \kappa \neq \emptyset$*, then the tree may be* $(<\kappa)$*-complete.*

(2) $\diamondsuit^{\Gamma}_{\lambda,1}$ *implies the existence of a* λ^+*-Souslin tree which remains non-special in any cofinalities preserving extension. Moreover, if* $\lambda^{<\kappa} = \lambda$ *and* $\Gamma \setminus \kappa \neq \emptyset$*, then the tree may be* $(<\kappa)$*-complete.*

The above generalizes and unify independent works of Jensen, Baumgartner and Cummings.

- Future collaboration with host institution (if applicable)
  - ▶ Stevo Todorcevic of the university of Paris 7 has asked whether it is consistent that all Aronszajn trees are special. By the work of Baumgartner, the above statement, restricted to trees of height ω_1, is indeed consistent. However, we believe that the general statement is inconsistent, and hope to prove that the existence of a special λ^+-Aronszajn tree implies the existence a *non-special* one, provided that $\lambda > \omega$.
  - ▶ Mirna Džamonja of the university of East Anglia pointed out that it is open whether the tree property may hold for all uncountable cardinals (at once). This raises the question whether, at least, the generalized continuum hypothesis implies $\diamondsuit^{\Gamma}_{\lambda,<\lambda^+}$ for **some** uncountable cardinal λ. This looks like a very hard question, though we conjecture an affirmative answer.
  - ▶ István Juhász of the Alfréd Rényi Institute of Mathematics asked whether it is provable in ZFC that there exists a λ^+-Souslin tree for **some** cardinal λ (including the case $\lambda = \omega$). This, in turn, would provide an answer to a topological question Juhász has been working on.
- Projected publications/articles resulting or to result from your grant

The long list of results will eventually be published in a paper whose title is, at least currently, "A unified approach to higher Souslin trees constructions".[1]

- Other comments (if any)

I am afraid that on my arrival to Paris, at the train station, my wallet has been stolen. This means that I do not have the physical boarding pass of my flight to Paris.

On the other hand, I do have the electronic boarding pass (because I have conducted "check-in" via the internet), and also other evidences to the fact that I arrived to Paris in the planned flight.

[1]Depending on its length, the paper may be split into two (e.g., "A unified approach to higher Souslin trees constructions, Part I" and "A unified approach to higher Souslin trees constructions, Part II").