Scientific Report

ESF short visit
"On precipitousness of normal ideals"

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I visited Professor Jacob Kellner at University of Vienna from September 2 to 19. We discussed normal precipitous ideals and precipitous towers of normal ideals. Mainly we studied ideals over ω_1 and countable towers of ideals.

It is well-known, due to Foreman, Magidor and Shelah [1], that if a supercompact cardinal is Lévy collapsed to ω_2, then NS_{ω_1} becomes precipitous. Moreover Ishiu [2] proved that normal ideals over ω_1 naturally defined from tail club guessing sequences are also precipitous in the same model. Recently I proved that normal ideals naturally defined from a variant of $\diamondsuit$-sequences are precipitous, too. We are interested in what kind of normal ideals over ω_1 become precipitous after a supercompact cardinal is Lévy collapsed to ω_2. We conjecture that it is related to the definability of normal ideals.

This question is closely related to precipitousness of countable tower of normal ideals up to a supercompact cardinal. The above mentioned fact on NS_{ω_1} corresponds to the fact that the countable tower of non-stationary ideals up to a supercompact cardinal is precipitous. Also, countable towers of normal ideals are naturally defined from tail club guessing sequences and a variant of $\diamondsuit$-sequences, and we can prove that these towers up to a supercompact cardinal are precipitous. We are also interested in what kind of towers of normal ideals up to a supercompact cardinal are precipitous.

During this short visit we investigated Shelah's note which claims that all reasonably definable normal ideals over ω_1 are precipitous after the Lévy collapse of a supercompact cardinal and that all reasonably definable towers of normal ideals up to a supercompact cardinal are precipitous. We also made a contact with Shelah by e-mails and phone calls and discussed his note with him.

As a result, we found a gap in his argument unfortunately. We also tried to amend it, but we could not do that during my visit. We will keep trying to amend it or to find a counter-example.

During my visit we could exchange our knowledge on precipitous ideals and precipitous towers of ideals, too, and it was very valuable for our future collaboration. I thank the ESF research networking program "New frontiers of infinity" so much for supporting my visit.

References

[1] M. Foreman, M. Magidor and S. Shelah, *Martin's maximum, saturated ideals, and nonregular ultrafilters I*, Ann. of Math. (2) **127** (1988), no.1, 1–47.

[2] T. Ishiu, *Club guessing sequences and filters*, J. Symbolic Logic **70** (2005), no.4, 1037–1071.