Final Report for Short Term Visit Grant

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April 13, 2011

1   Overview

The Young Set Theory 2011 Workshop was held in Königswinter between March 21-25, 2011. With the generous support from The ESF Research Networking Programme INFTY and kind host from organization commitee, I was able to participate in the workshop. The aim of the workshop is to bring together young researchers in set theory in order to share knowledge and to exchange thoughts. During the workshop, I attended several wonderful research talks and also made deep discussions with other colleagues which lead to many new ideas for my further research.

2   Tutorial and Research Talks

There are several tutorials and research talks together with plenty of special discussion sections from which I benefit a lot.

Joel David Hamkins gave a tutorial in Boolean ultrapowers. He described a new interpretation of some common knowledge which leads to surprising set theory structure. He also indicted the possibility of a new characterization of generic ultrapower in terms of Boolean ultrapower which seems to be possible to replace the usage of precipitous ideal in some stage.

Assaf Rinot gave a talk on his Ostaszewski square and relative consistency result. His talk covered almost all the question and application regarding square sequence. By adapting Ostaszewski square in the minimal walk technique, he gave a new construction of strong coloring.

I also learn a lot from discussion section led by Vincenzo Dimonte and Matteo Viale. Vincenzo Dimonte talked about large cardinals above I0 which is very interesting to audience and he also point out the possibility and obstacle for combining forcing with such kind large cardinal. Matteo Viale talked about ITP and guessing models which are large cardinal properties in the region of supercompactness. He gave a proof that the localization of such properties can be derived from PFA which suggest that PFA has very large consistency strength.

3 Research Progress

3.1 Definability of Ultrafilter

After a sequel of discussion with Radek Honzik, I achieved great process in the problem described as following.

It is well-known that any ultrafilter U on κ cannot be definable over the structure $\langle H(2^\kappa), \in \rangle$ in any parameter. So a natural question is whether there is a ultrafilter definable over structure $H(2^{\kappa^+})$. It is clear that in canonical inner model $L[U]$ for one measurable cardinal κ, U is the unique ultrafilter on κ such that U is Σ_1^1 lightface definable in $\langle H(\kappa^{++}), \{P(\kappa)\}, \in \rangle = \langle H(2^{\kappa^+}), \{P(\kappa)\}, \in \rangle$. However, the question remain unsolved when we require $2^\kappa > \kappa^+$.

I am able to prove that:

Theorem 3.1. *Assume the consistency of ZFC+$o(\kappa) = \kappa^{++}$, it is consistent that ZFC+$2^\kappa = \kappa^{++}$+there is a normal ultrafilter U on κ which is Σ_1^1 definable over $\langle H(\kappa^{++}), \in \rangle$.*

Furthermore, it is possible to make U be the unique normal ultrafilter on κ.

3.2 Forcing and Condensation

During the workshop, I exchange opinions and results on the specified topic regarding forcing and condensation with colleagues. Inspired from the discussion, I am able to make a small progress to show that assume that $0^\sharp$ does not exists, then there are small forcing force Strong Condensation for $H(\aleph_n)$.

It is also very impressive to learn from Assaf Rinot's talk that his Ostaszewski Square Principle is equivalent to Jensen's Square Principle under GCH. So if it is possible to prove Jensen's Square Principle from Axiom of Strong Condensation, more application of Axiom of Strong Condensation will come out.