Scientific reports

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1. Purpose of the visit

   This workshop is unique in the sense that many young researchers in set theory gather in one place and exchange their topics and ideas in set theory. I would like to learn many things from them and broaden my view of set theory while discussing my main topic with them.

2. Description of the work carried out during the visit

   I discussed with Grigor Sargsyan on extension of A-closed structures to models of ZFC with the same reals, which would give us an evidence of the equivalence between Ω-conjecture and the completeness "theorem" of 2nd-order Boolean-valued logic on which I am currently working with Jouko Väänänen.

   I also attended lectures and discussion sessions among which Matteo Viale's work on guessing models, Sean Cox's diagonal reflection principle, and Joel Hamkins' Boolean ultrapower are interesting to me.

3. Description of the main results obtained

   Grigor Sargsyan has observed the following:

   Theorem 1. Suppose there is a proper class of Woodin cardinals and let A be a universally Baire set of reals. Then there is a universally Baire function $f : \mathbb{R} \to \mathbb{R}$ such that for any τ closed under f there is a transitive model M of ZFC such that $M \cap \mathbb{R} = \tau$ and M is A-closed.

   This theorem would give us an plausible scenario for the following conjecture:

Conjecture 1. Suppose there is a proper class of Woodin cardinals. Then Ω-conjecture is equivalent to the Completeness "Theorem" for the 2nd-order Boolean-valued logic.

Currently I am working with Jouko Väänänen on the connection between Ω-logic and 2nd-order Boolean-valued logic. We showed that the validity predicate of Ω-logic is as complex as the one of 2nd-order Boolean-valued logic, in particular, assuming the Ω-conjecture, the complexity of the validity predicate of 2nd-order Boolean-valued logic is Δ_2 in the language of set theory while the one for the full 2nd-order logic is Π_2-complete. This result lead us to formulating the syntax of 2nd-order Boolean-valued logic and stating the above conjecture. The above theorem would be the key point towards proving the above conjecture.

4. Projected publications/articles resulting or to result from the grant.

   The result in this workshop will be in the paper I am going to write with Jouko Väänänen on the connection between Ω-logic and 2nd-order Boolean-valued logic.