Project report: percolation in the contact process

The purpose of this project was to study percolation properties of the 'upper invariant measure' of the contact process, especially a variant of the contact process inspired by a recent article[1] which studies models for the spread of vegetation in arid regions.

The contact process on $\mathbb{Z}^d$ is a Markovian model for the spread of disease, which for sufficiently high infection rates has an invariant measure with a positive density of infected individuals. It is known that (for sufficiently high infection rates) the infected sites percolate under this measure.

We consider more general models in which there is an additional *spontaneous* infection rate. Numerical simulations performed in the paper referred to above suggest that it may be particularly interesting to study cases where this spontaneous infection rate is a function of the instantaneous overall density of infected sites. The authors interpret their results as indicating a form of 'self-organized criticality', a type of behaviour which is very different from that in 'ordinary' percolation models.

The main aim of the visit was to rigorously study the percolation properties in such variants of the contact process, in order to (a) shed light on the simulation results of the paper in *Nature*, and (b) generally try to get a better understanding of the percolation properties of contact processes. Here follows a description of the main results obtained during the visit.

Firstly, it is not a priori clear that it is possible to define a contact process whose spontaneous infection rate depends on the instantaneous density. However, using time discretization, a coupling argument, and graphical representation we were able to construct a class of processes of this type. Next, we were able to deduce a simple correspondence between the invariant measures of such density-dependent processes and those of the better studied processes with constant spontaneous infection rate. Expressing this correspondence in terms of the solutions to a certain fixed-point equation, we were able to deduce that the type of self-organized criticality referred to above would imply a similar property of the process with constant spontaneous infection rate. Moreover, in the case of 2 spatial dimensions we made substantial progress on proving a sharpness result which would rule out the self-organized criticality referred to above. The arguments for this are based on recent work by Prof. van den Berg; completing the final details is ongoing work.

We consider the results described above interesting in their own right. However, further interest is gained through their applicability to the paper in *Nature* mentioned earlier. To make our conclusions even more relevant to that work, we are currently studying a 3-state version of the contact process which is closer to the process studied there. We strongly believe that the long-term behaviour of this 3-state process is sufficiently similar to that of the more familiar 2-state process that the results described above can be applied more or less directly. This is also ongoing work.

We regard the visit as highly productive and successful, and expect that our results and ongoing work will lead to a publication.

[1] Kéfi et al.: 'Spatial vegetation patterns and imminent desertification in mediterranean arid ecosystems'. *Nature*, 449, 2007.