Report of an ITGP-ESF short visit to Luxembourg 16th-21st of May 2011

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Research Report

1 Purpose of the visit

The aim of this project was to find the good definition of the notion of "rackoid", which is the object which should integrate Leibniz algebroids. Motivations for such objects come for example from Courant algebroids, which are in particular Leibniz algebroids with additional structure. Leibniz algebra (non antisymmetric version of Lie algebras) are special examples of Leibniz algebroids over a point and are already known to integrate to racks.

Hinich and later Getzler and Henriques have introduced a process of integration of graded Lie algebras, or more generally homotopy Lie algebras, which is based on the ideas of topological realization of Sullivan. Basically they view a graded Lie algebras on V as a homological derivation δ of the symmetric algebra $S^\bullet(V[1])$. In particular $(S^\bullet(V[1]), \delta)$ forms a differential graded commutative algebra, and therefore one can consider its Sullivan's topological realization $Hom_{DGCA}(S^\bullet(V[1]), \Omega^\bullet(\Delta_n))$ which is known to be a Kan simplicial set. This object can be understood as integrating the given homotopy Lie algebra, since by a result of Grothendieck, nerves of Lie groupoids are precisely the Kan simplicial sets whose horn fillers are bijections for $n > 1$.

Our strategy was to give a non-commutative version of this theory : a Leibniz algebra on V can be seen as a homological derivation δ of the Zinbiel algebra $T^\bullet(V[1])$ which together form a Differential Graded Zinbiel Algebra (non commutative). Therefore we wanted to develop a non commutative version of Sullivan's topological realization which should produce a Kan cubical set. This notion of Kan cubical set was not well defined, but we had already a candidate. By taking Kan cubical sets such that the horn fillers are bijections for $n > 1$, we expected to get the axioms for the rackoids we were looking for.

2 Work carried out during the visit

We have first studied the algebraic structures coming from the compatibility conditions generated by the filling of the 3-dimensional cubical sets. We found this way 6 constraints which turned out to be non equivalent. Among them one gives the axioms of a left rack we were looking for and another one gives the axioms of a right rack. It is an open question whether the 4 other types of structure are interesting. We have also tried to have a confirmation of the fact that our notion of Kan cubical set is meaningful, i.e. to find natural examples which should be cubic analogues of simplicial examples. Therefore we searched for the nerve of a cubical anologue of the notion of category. It turns out that this cubical analogue of a category should be the two skeleton of a Kan cubical set, so in a sense, this leads to a logical short-cut in our strategy. This is why we have switched to the other potential source of examples, namely by the analogue of Sullivan's topological realization for Leibniz algebras.

3 Main results obtained

The main result of the research carried out during the visit was the appearance of 4 new structures and the recovery of left and right racks. The infinitesimal version of these structures was computed

and led, among others, to a generalization of Leibniz algebras. The impossibility to integrate Leibniz algebroids became clear since this would need a notion of "conjugation" on the source and the target of an arrow by a same element which would need a kind of parallel transport which does not exist in general. However the new structures we have found have a meaning for cubical sets over more than a point, so this gives a way to to give a notion which integrates not the notion of Leibniz algebroids, but of a little more general structure involving an extra product. When restricted to cubical sets with only one object, and when this product acts trivially, we recover the usual notion of a rack.

4 Future collaboration

We plan to continue the study of the Sullivan's approach for Leibniz algebras to find examples of Kan cubical sets and validate our definition. We plan to meet again one week in June (in Luxembourg) and one week in Lens in July.

5 Projected publications to result from the grant

If the study of the Sullivan approach for Leibniz algebras gives a Kan cubical set in our sense, we plan to write our results in an article, and submit it to a journal specialized in algebraic topology or category theory.