Report of an ITGP-ESF short visit to Luxembourg 21st- 25th of June 2011

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Research Report

1 Purpose of the visit

As a follow up of the previous visit, we recall :

" The aim of this project was to find the good definition of the notion of "rackoid", which is the object which should integrate Leibniz algebroids. Motivations for such objects come for example from Courant algebroids, which are in particular Leibniz algebroids with additional structure. Leibniz algebra (non antisymmetric version of Lie algebras) are special examples of Leibniz algebroids over a point and are already known to integrate to racks.

Hinich and later Getzler and Henriques have introduced a process of integration of graded Lie algebras, or more generally homotopy Lie algebras, which is based on the ideas of topological realization of Sullivan. Basically they view a graded Lie algebras on V as a homological derivation δ of the symmetric algebra $S^\bullet(V[1])$. In particular $(S^\bullet(V[1]), \delta)$ forms a differential graded commutative algebra, and therefore one can consider its Sullivan's topological realization $Hom_{DGCA}(S^\bullet(V[1]), \Omega^\bullet(\Delta_n))$ which is known to be a Kan simplicial set. This object can be understood as integrating the given homotopy Lie algebra, since by a result of Grothendieck, nerves of Lie groupoids are precisely the Kan simplicial sets whose horn fillers are bijections for $n > 1$.

Our strategy was to give a non-commutative version of this theory : a Leibniz algebra on V can be seen as a homological derivation δ of the Zinbiel algebra $T^\bullet(V[1])$ which together form a Differential Graded Zinbiel Algebra (non commutative). Therefore we wanted to develop a non commutative version of Sullivan's topological realization which should produce a Kan cubical set. This notion of Kan cubical set was not well defined, but we had already a candidate. By taking Kan cubical sets such that the horn fillers are bijections for $n > 1$, we expected to get the axioms for the rackoids we were looking for."

2 Work carried out during the visit

We have studied the Sullivan's approach and its application to integration of L_∞ algebras as exposed in Getzler's article. In order to transpose it to Leibniz algebras, we have started to study the work of Muriel Livernet who has developed a version of rational homotopy theory for Leibniz algebras. But we have realized that some piece of the puzzle is still missing to define the topological realization functor, namely the Leibniz analog of $\Omega^\bullet(\Delta_n)$.

3 Main results obtained

The main result of the research carried out during the visit is a better understanding of Sullivan's techniques and the state of the art in of its generalization to the Leibniz case, together with a clear identification of what is missing to achieve our goal. This can seem deceiving compared to our original motivation (cf above), but has to be balanced with the time devoted to this project for the moment (two weeks)

4 Future collaboration

We plan to continue the study of the Sullivan's approach for Leibniz algebras to find examples of Kan cubical sets and in particular to find the good notion of topological realization functor. We plan to continue our collaboration in Fall in Luxembourg and in Lens.

5 Projected publications to result from the grant

If the study of the Sullivan approach for Leibniz algebras gives a Kan cubical set in our sense, we plan to write our results in an article, and submit it to a journal specialized in algebraic topology or category theory.