Prof. Alexander I. Nosich

SCIENTIFIC REPORT FOR SHORT VISIT
to Ecole Normale Superieur de Cachan and University of Rennes 1 in November 2011
in the framework of the ESF Network "Newfocus"

Topic: *"Analytical regularization and Nystrom-type methods in the modeling of dielectric lenses and resonators"*

Purpose of the visit to Cachan: collaboration with Dr. M. Lebental

Description of the work carried out during the visit from 13th to 20th November 2011:
- seminar presentation at the ENS de Cachan related to the topic indicated above with application to modelling of emission from microcavity lasers; the seminar was attended by Dr. M. Lebental, Prof. J. Zyss, students, and other staff members,
- discussions with Dr. M. Lebental and Prof. J. Zyss of the topics of common interest; an agreement was achieved that the emission of waves from the thin flat dielectric laser cavities shaped as a kite can be an area of collaboration, with electromagnetic-field modeling done in Kharkov and experimental measurements done in Cachan.

Purpose of the visit to Rennes: collaboration with Professor R. Sauleau

Description of the work carried out during the visit from 21st to 28th November 2011:
- discussions with Prof. Sauleau of the ongoing collaboration efforts around the two-dimensional (2-D) and three-dimensional (3-D) modeling of lenses and discrete scatterers arranged as periodic arrays of electromagnetically coupled sub-wavelength metal and dielectric wires and strips,
- work on two joint papers to be submitted to the well-reputed international technical journals, *IEEE Antennas and Propagation* and *IEEE Photonics Technology Letters*.

Details of the topics discussed in Cachan and Rennes:
Dielectric open resonators find numerous applications in science and technology across wide spectrum of wavelengths from millimetre to THz waves and optics. Microcavity lasers of the THZ and infrared ranges such as those studied in Cachan can be also modelled as dielectric resonators however with active regions able to generate electromagnetic waves under pumping. Lenses are also ubiquitous components in short-wave electromagnetic sub-systems and systems. Although their basic design principles are suggested by geometrical optics, any finite-size lens has a finite-size focal spot whose size and shape depends on the wavelength. Still any real lens is simultaneously an open resonator, whose natural modes can spoil the lens focusing ability.

Mathematical and numerical analysis of dielectric scatterers is a challenging problem, the widely recognized underlying difficulty being a combination of the ray-like and the mode-like features in the electromagnetic field behaviour. Numerous attempts have been done to use Geometrical Optics to study the ray dynamics in 2-D dielectric cavities, including the works of the group in Cachan. Although this approach has led to several important discoveries such as "bow-tie" resonances in the stadium-shape and elliptic dielectric resonators, it fails to grasp the discreteness of modes and provide adequate evaluation of the radiation losses.

As a well-grounded alternative, several integral-equation (IE) formulations have been proposed, where the Muller equations stand aside as the Fredholm second kind IEs. A discretization scheme based on a Galerkin method with trigonometric polynomials as basis functions can be applied to obtain an infinite-matrix equation with favourable features. It exploits the possibility of exact solution of the scattering by a circular cylinder (in 2-D) or a sphere (in 3-D), i.e. the analytical regularization. Such an approach has been used by the Kharkov team since the 1990s.

Besides, the same Muller IEs can be directly discretized even more efficiently using a Nystrom algorithm and corresponding quadrature formulas for numerical integration. Such an approach has been under active development by the Kharkov team since the late 2000s.

By using Muller IEs discretized with Galerkin or Nystrom techniques, the study of resonances is reduced to the search of roots of determinantal equations. The advantages of both algorithms that make them efficient and reliable tools for the analysis of dielectric scatterers are as follows: (i) controllable accuracy, i.e., possibility to minimize the computational error to a predicted number of digits for an arbitrary set of the problem parameters (i.e. dielectric scatterer size, shape and material) by solving progressively greater matrices; (ii) low memory and time requirements; (iii) stability near and far from the sharp natural resonances, such as whispering-gallery modes, which is hard with conventional numerical approximations; and (iv) absence of the false "numerical resonances" intrinsic to the IEs methods not based on the Muller IEs.

Numerical results on the natural modes of dielectric resonators and the wave scattering by discrete dielectric scatterers have been presented. In Cachan, these examples were concentrated at the two-dimensional kite-shape dielectric resonators, which are able to provide highly directive far-zone emission patterns. In Rennes, discussions were centred on discrete albeit periodic structures able to concentrate the field in sub-wavelength domains and on the perspectives of 3-D modelling of dielectric resonators and lenses with Muller IEs.