Report for short visit RGLIS/4446

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November 17, 2011

1 Purpose

In the wake of the first RGLIS sponsored short visit (RGLIS/4076) I derived a new sufficient condition for the of convergence of the cluster expansion of hard-core lattice gases / polymer system. The primary aim of this visit was to verify its proof. We also wanted to discuss comparisons with other conditions and applications to particular cases and related settings.

2 Results

The primary aim is fulfilled - the new condition seems to be correct (as far as 4 eyes can see). It strictly improves on an improved inductive Dobrushin-style [Dob96] condition [SS05] by taking into account the structure of the neighbourhood of a location/polymer. The improvement comes from a use of tree-expansion methods [FP07]. The difference to [FP07] lies in the choice of a novel partition scheme – thus the two conditions are not directly comparable. Higher-depth versions of those conditions give the same limit, that is the exact domain of convergence, though. The problem is that they are quite impossible to calculate explicitely for $k \geq 3$ and difficult to use.

It seems that the new criterion is better on systems with low degree (coordination number) in the homogeneous case. It also allows to derive the classic Gruber-Kunz condition for contours [GK71]. We also tried to apply it to the hard-sphere gas [Rue69, FPS07], but quickly found that the necessary multi-dimensional integrals take more than just one afternoon.

In the end we explored some possibilities of finding a partition scheme combining both approaches and thus a condition improving on both the [FP07] and the new condition.

3 Outlook and comments

I want to publish this result. The current focus is to include it in my thesis and proceed with other work in spring 2012 after my defense.

The same holds for the applications, which require some computational effort. The idea of uniting the approach by [FP07] with the present one requires conforming to the invariants of both approaches in a harmonious way – I conjecture that this is possible but needs some quiet time to write it down and verify that I missed nothing.

One thing that I want to do as part of my thesis work is to evaluate both conditions on some sample graphs to get a better understanding of where their respective strengths lie.

Finally I wand to add that it has been nice to show verifiable progress and on the same time learn on the side a lot about the bigger picture and questions in the statistical mechanics community.

References

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