Scientific report on a Short Visit Grant in the Framework of the ESF Research Networking Programme NEWFOCUS

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Period of Stay: 23 November – 5 December 2011, 13 days

Project Title: **Nonlinear effects in the scattering of millimeter, sub-millimeter and terahertz waves by finite layered structures**

Purpose of the visit

The main objective of this project was to explore the properties of electromagnetic waves in nonlinear multilayered quasi-periodic structures comprised of dielectric layers. The self-consistent theoretical models, taking into account nonlinear properties of dielectric layers have been developed for characterization of electromagnetic wave propagation in these structures. The harmonic and combinatorial frequency generation by the quasi-periodic structure composed of alternating layers of two nonlinear dielectrics has been examined. The three-wave interaction technique has been used to study the nonlinear processes in the finite structure illuminated by the plane waves of two tones. The properties of the combinatorial frequency waves emitted from the stacked nonlinear layers in millimeter and THz frequency ranges has been discussed.

Description of the work carried out during the visit

Within the framework of the project we consider a finite periodic structure formed by two alternating dielectric layers of thicknesses d_1 (layer A) and d_2 (layer B). The layers A and B alternate in the quasi-periodic Fibonacci sequence. A Fibonacci system is based on the recursive relation $S_1 = \{A\}$, $S_2 = \{AB\}$ and $S_q = \{S_{q\text{-}1}\, S_{q\text{-}2}\}$ for $q \geq 2$ (here q is the Fibonacci number). We assume that L is the total thickness of the multilayered structure. For the Fibonacci structure $L_1 = d_1,\ L_2 = d_1 + d_2,\ L_q = L_{q-1} + L_{q-2}$. The geometry of the problem is shown in Fig.1. The stack of nonlinear layers is surrounded by the linear homogeneous medium with dielectric permittivity ε_a at $z \leq 0$ and $z \geq L$. Assuming that two waves of frequencies ω_1 and ω_2 are incident at angles

Θ_{i1} and Θ_{i2} on the surface of the nonlinear dielectric structure, we analyze the generation of a wave at the combinatorial frequency $\omega_3=\omega_1+\omega_2$.

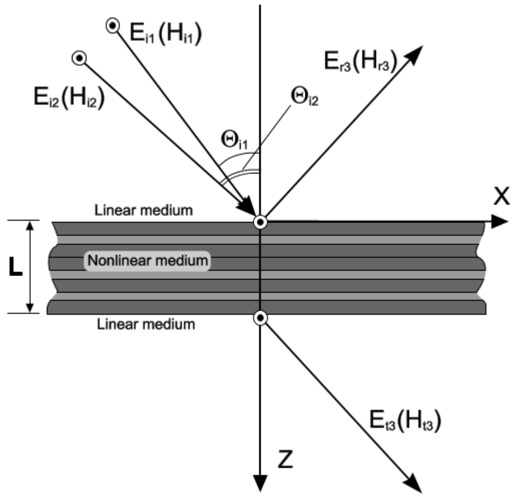

Fig.1 Geometry of the problem.

The nonlinear dielectric layers have class 6mm anisotropy, which is typical for the uniaxial crystals of CdS, CdSe, ZnO, α-ZnS, and AgI. The optical axes of all layers are oriented along the z-direction as shown in Fig. 1, and each layer is described by tensors of linear dielectric permittivity $\hat{\varepsilon}=(\varepsilon_{xx},\varepsilon_{xx},\varepsilon_{zz})$ and nonlinear susceptibility $\hat{\chi}$

$$\hat{\chi}=\begin{pmatrix} 0 & 0 & 0 & 0 & \chi_{xxz} & 0 \\ 0 & 0 & 0 & \chi_{xxz} & 0 & 0 \\ \chi_{zxx} & \chi_{zxx} & \chi_{zzz} & 0 & 0 & 0 \end{pmatrix}.$$

Since the structure is spatially homogeneous in the *x*- and *y*- directions, the problem can be reduced to the independent analyses of the TE and TM waves without field variations along the *y*-axis, i.e. $\partial/\partial y=0$. In this work, we will consider only the TM waves with the field components E_x, E_z, H_y, while treatment of the TE waves is similar and somewhat simpler being unaffected by anisotropy of $\hat{\chi}$.

To calculate the coefficients of reflection and transmission the transmission matrix method is used. It connects the field at the beginning and at the end of the layer in a general case, the transmission matrix of the whole system is obtained by multiplying the individual layer matrices $\hat{m}_i$

$$\hat{M}=\prod_i \hat{m}_i$$

according to a chosen sequence. For the case of Fibonacci quasiperiodic structure

$$\hat{M}_1=\hat{m}_1,\ \hat{M}_2=\hat{m}_1\hat{m}_2,\ \hat{M}_q=\hat{M}_{q-1}\hat{M}_{q-2}(q>2)$$

Here $\hat{m}_{1,2}$ are the matrixes of the layers A and B.

Assume that $k_x = \frac{\omega}{c}\sqrt{\varepsilon_a}\sin\theta_i$, the transversal wave numbers for the homogeneous media are $k_{za} = \frac{\omega}{c}\sqrt{\varepsilon_a}\cos\theta_i$, where θ_i is the angle of electromagnetic wave incidence. Using the boundary conditions for tangential components of the electromagnetic field at $z=0$ and $z=L$, we arrive the expressions for reflection and transmission coefficients for electromagnetic wave

$$R(\omega) = \frac{M_{q11}(\omega) + \frac{c}{\omega}\frac{k_{za}(\omega)}{\varepsilon_a}M_{q12}(\omega) - \frac{\omega}{c}\frac{\varepsilon_a}{k_{za}(\omega)}M_{q21}(\omega) - M_{q22}(\omega)}{M_{q11}(\omega) + \frac{c}{\omega}\frac{k_{za}(\omega)}{\varepsilon_a}M_{q12}(\omega) + \frac{\omega}{c}\frac{\varepsilon_a}{k_{za}(\omega)}M_{q21}(\omega) + M_{q22}(\omega)}$$

$$T(\omega) = \frac{2e^{-ik_{za}(\omega)L}}{M_{q11}(\omega) + \frac{c}{\omega}\frac{k_{za}(\omega)}{\varepsilon_a}M_{q12}(\omega) + \frac{\omega}{c}\frac{\varepsilon_a}{k_{za}(\omega)}M_{q21}(\omega) + M_{q22}(\omega)}$$

The wave equation for the fields of the combinatorial frequency ω_3 in the nonlinear dielectric slabs d_1 takes the form

$$\frac{\partial^2 H_y^A(\omega_3)}{\varepsilon_{xx1}\partial z^2} + \left(k_3^2 - \frac{k_{x3}^2}{\varepsilon_{zz1}}\right)H_y^A(\omega_3) = 4\pi k_3\left[2\frac{\partial}{\partial x}\left(\frac{\chi_{zxx1}}{\varepsilon_{zz1}}E_x^A(\omega_1)E_x^A(\omega_2) + \frac{\chi_{zzz1}}{\varepsilon_{zz1}}E_z^A(\omega_1)E_z^A(\omega_2)\right) - \right.$$
$$\left. - \frac{\chi_{xxz1}}{\varepsilon_{xx1}}\frac{\partial}{\partial z}\left(E_x^A(\omega_1)E_z^A(\omega_2) + E_x^A(\omega_2)E_z^A(\omega_1)\right)\right],$$

where $k_3 = \omega_3/c$ and $k_{x3} = k_3\sqrt{\varepsilon_a}\sin\Theta_3$ is determined by the requirement of the phase synchronism in the three-wave mixing process

$$k_{x3} = k_{x1} + k_{x2},$$

where $k_{x1,x2} = k(\omega_{1,2})\sqrt{\varepsilon_a}\sin\Theta_{i1,i2}$. The solutions of wave equation include partial solution of the inhomogeneous equation and general solution of the homogeneous one

$$H_y^A(\omega_3) = \left(A_1^+ e^{ik_{zL1\_3}z} + A_1^- e^{-ik_{zL1\_3}z} + N_1^+ e^{ik_{zL1}^+ z} + N_2^+ e^{-ik_{zL1}^+ z} + N_1^- e^{ik_{zL1}^- z} + N_2^- e^{-ik_{zL1}^- z}\right)e^{-i\omega_3 t + ik_{x3}x}$$

Here $A_1^\pm$ are the amplitude coefficients in the solution of the homogeneous equation, and they are obtained from the continuity conditions for the tangential field components. The other four terms arise from the solution of the inhomogeneous equation, coefficients $N_{1,2}^\pm$ are expressed in terms of the field magnitudes in the layer at frequencies $\omega_{1,2}$

$$N_1^+ = \alpha\beta\frac{B^+(\omega_1)B^+(\omega_2)}{\left(k_{z\_A}^+\right)^2 - \left(k_{zL3}\right)^2},\quad N_2^+ = \alpha\beta\frac{B^-(\omega_1)B^-(\omega_2)}{\left(k_{z\_A}^+\right)^2 - \left(k_{zL3}\right)^2}$$
$$N_1^- = \alpha\gamma\frac{B^+(\omega_1)B^-(\omega_2)}{\left(k_{z\_A}^-\right)^2 - \left(k_{zL3}\right)^2},\quad N_2^- = \alpha\gamma\frac{B^-(\omega_1)B^+(\omega_2)}{\left(k_{z\_A}^-\right)^2 - \left(k_{zL3}\right)^2}$$

$$\alpha = \frac{4\pi}{\varepsilon_{zz}}\frac{k_3}{k_1k_2},\ \beta = -k^{+}_{z\_A}k_{x1}k_{zL2}\frac{\chi_{xxz}}{\varepsilon_{xx}} - k_{x3}\left(\frac{\chi_{zxx}}{\varepsilon_{xx}}k_{z\_A\_1}k_{z\_A\_2} + \frac{\chi_{zzz}\varepsilon_{xx}}{\varepsilon^2_{zz}}k_{x1}k_{x2}\right),$$

$$\gamma = k^{-}_{z\_A}k_{x1}k_{zL2}\frac{\chi_{xxz}}{\varepsilon_{xx}} + k_{x3}\left(\frac{\chi_{zxx}}{\varepsilon_{xx}}k_{z\_A\_1}k_{z\_A\_2} - \frac{\chi_{zzz}\varepsilon_{xx}}{\varepsilon^2_{zz}}k_{x1}k_{x2}\right),$$

$$k_{z\_A\_1, z\_A\_2, z\_A\_3} = \sqrt{\left(k^2_{1,2,3} - \frac{k^2_{x1,x2,x3}}{\varepsilon_{zz}}\right)\varepsilon_{xx}}\ ;\ k_{1,2} = \omega_{1,2}/c$$

The boundary conditions at the layer interfaces require that

$$k^{\pm}_{z\_A} = k_{z\_A\_1} \pm k_{z\_A\_2}$$
$$k^{\pm}_{z\_B} = k_{z\_B\_1} \pm k_{z\_B\_2}.$$

The transfer matrix method can now be applied to interrelating the fields of combinatorial frequency ω_3 at the layer interfaces. Then the field continuity conditions require that

$$\begin{pmatrix} H^A_{y3}(0) \\ E^A_{x3}(0) \end{pmatrix} = \hat{m}_1(\omega_3)\begin{pmatrix} H^A_{y3}(d_1) \\ E^A_{x3}(d_1) \end{pmatrix} + \hat{m}_1(\omega_3)\begin{pmatrix} \tau_1(d_1) \\ \xi_1(d_1) \end{pmatrix}$$

where $\tau_1(d_1)$ and $\xi_1(d_1)$ are contain the terms proportional to the coefficients $N^{\pm}_{1,2}$.

The inhomogeneous wave equation for $H^B_y(\omega_3)$ has the same form and the amplitudes of $H^B_y(\omega_3)$ are similar to those in layer d_1. Then

$$\begin{pmatrix} H^B_{y3}(d_1) \\ E^B_{x3}(d_1) \end{pmatrix} = \hat{m}_2(\omega_3)\begin{pmatrix} H^B_{y3}(d_1+d_2) \\ E^B_{x3}(d_1+d_2) \end{pmatrix} + \hat{m}_2(\omega_3)\begin{pmatrix} \tau_2(d_1+d_2) \\ \xi_2(d_1+d_2) \end{pmatrix}$$

Using the transfer matrix method and the field continuity condition we can connect the fields of frequency ω_3 at the interfaces of the investigated structure. For example in the case of structure S_4 we obtain

$$\begin{pmatrix} H^A_{y3}(0) \\ E^A_{x3}(0) \end{pmatrix} = \hat{M}_4(\omega_3)\begin{pmatrix} H^B_{y4}(L) \\ E^B_{x4}(L) \end{pmatrix} + \hat{M}_1(\omega_3)\begin{pmatrix} \tau_1(L_1) \\ \xi_1(L_1) \end{pmatrix} + \hat{M}_2(\omega_3)\begin{pmatrix} \tau_2(L_2) \\ \xi_2(L_2) \end{pmatrix} + \hat{M}_3(\omega_3)\begin{pmatrix} \tau_1(L_3) \\ \xi_1(L_3) \end{pmatrix} +$$
$$+\hat{M}_3(\omega_3)\left(\hat{M}_1(\omega_3)\begin{pmatrix} \tau_1(L_3+L_1) \\ \xi_1(L_3+L_1) \end{pmatrix} + \hat{M}_2(\omega_3)\begin{pmatrix} \tau_2(L_4) \\ \xi_2(L_4) \end{pmatrix}\right)$$

The amplitudes $F_{r,t}$ of the waves scattered from the layer into the surrounding linear media (F_r at $z<0$ and F_t at $z>L$) at the combinatorial frequency ω_3 are determined by enforcing the boundary conditions of $H_y(\omega_3)$ and $E_x(\omega_3)$ continuity at the layer interfaces $z = 0, L$.

Based on the solution of the outlined problem, the intensities of the waves of frequency ω_3 outside the finite photonic crystal and the fields localized inside the structure are explored. The numerical simulation was done for the multilayered structure with the following parameters: $\varepsilon_{xx1} = 5.382$, $\varepsilon_{zz1} = 5.457$, $\chi_{xxz1} = 2.1\times10^{-7}$, $\chi_{zxx1} = 1.92\times10^{-7}$, $\chi_{zzz1} = 3.78\times10^{-7}$, $\varepsilon_{xx2} = 1.4$, $\varepsilon_{zz2} = 2.6$, $\chi_{xxz2} = 2.82\times10^{-8}$, $\chi_{zxx2} = 2.58\times10^{-8}$, $\chi_{zzz2} = 8.58\times10^{-8}$. The numerical simulation for plane-wave reflection from the quasiperiodic multilayered structures is presented in Fig.2. The results are presented for three different thicknesses of the stack (variable Fibonacci numbers).

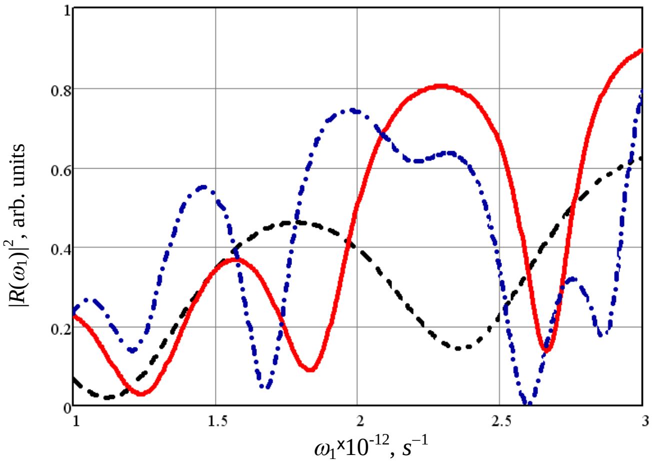

Fig. 2. Reflectance of TM wave incident at Θ_i = 30° on the quasiperiodic stack of dielectric layers of thicknesses d_1=0.1 mm, d_2=0.07 mm at q=4 (black dashed curve), q=5 (red solid curve) and q=5 (blue hatched curve) versus frequency

It can be seen, that for the investigated frequency range full reflectance increases with increasing of the structure thickness.

The dependence of the intensity $|F_r|^2$ of the field at the combinatorial frequency $\omega_3 = \omega_1 + \omega_2$, emitted from the fine-layered periodic structure into surrounding media with equal permittivities ε_a=ε_b=1, is presented in Fig.3. It is shown that the intensity of the field emitted from the nonlinear photonic structure at combinatorial frequency ω_3 significantly varies with the number of layers or Fibonacci number too. Inspection of these plots shows that the maxima of $|F_{r,t}|^2$ and minima of the pump wave reflectance are correlated. However, it was demonstrated that $|F_t|^2$ is some times larger than $|F_r|^2$ while average levels of both $|F_{r,t}|^2$ grow with the structure thickness L.

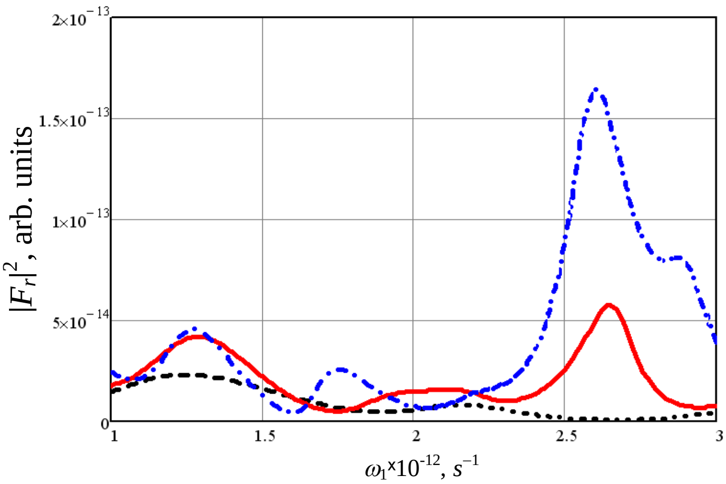

Fig. 3. Intensity of the wave at frequency ω_3 radiated from the periodic layered structure in the reverse direction of the z-axis at Θ_{i1} = 30°, Θ_{i2} = 45°; ω_2 = 0.8? 10^{12} s^{-1} and d_1=0.1 mm, d_2=0.07 mm at q=4 (black dashed curve), q=5 (red solid curve) and q=5 (blue hatched curve) versus frequency of the first pump wave

Intensity of the field radiated from the stack at frequency ω_3 dramatically increases when the incidence angles of the pump waves differ, $\Theta_{i1} \neq \Theta_{i2}$. It was shown that the magnitude of the mixing products $|F_{r,t}|^2$ scattered at the combinatorial frequency ω_3 is strongly correlated with the incidence angles of the pump waves. Moreover the field strength of the pump waves in the structure depends on their reflection $R(\omega_{1,2})$ and refraction $T(\omega_{1,2})$ coefficients which vary with frequency, angles of incidence $\Theta_{i1,2}$, structure thicknesses and the anisotropy of the permittivity $\hat{\varepsilon}$. Anisotropy of the nonlinear susceptibility $\hat{\chi}$ also influences the phase synchronism which determines the efficiency of the mixing process. Finally, it is necessary to note that dissipation losses in the layers alter not only magnitude of the $|F_{r,t}|^2$ peaks but also their frequencies. The dependences of the intensities of the waves at the combinatorial frequency on the dissipation processes have been analysed. The dependencies $|F_{r,t}|^2(\omega_1)$ for the tg $\Delta_{xx,zz}$=0.01 are shown in Fig.4. It should be noted that with allowance of dissipation intensity of the reflected wave at combinatorial frequency is more dependent on the dissipation processes. When tg $\Delta_{xx,zz} \neq 0$ the peaks of $|F_r|^2$ and $|F_t|^2$ are not reached simultaneously.

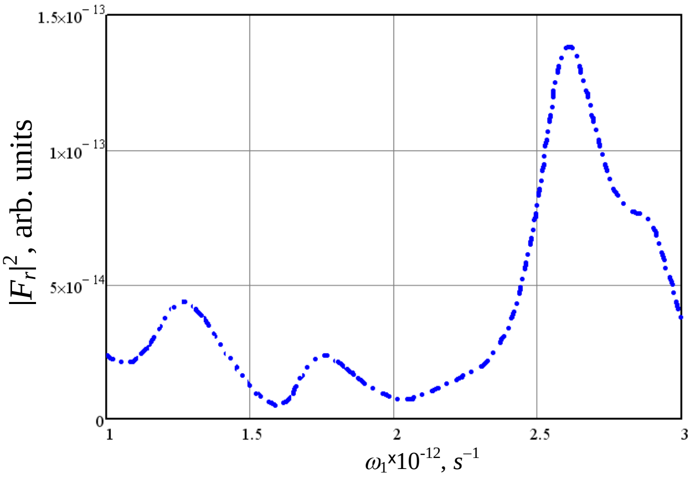

Fig. 4. Intensity of the wave at frequency ω_3 radiated from the periodic layered structure in the reverse direction of the z-axis at Θ_{i1} = 30°, Θ_{i2} = 45°; ω_2 = 0.8?10^{12} s^{-1} and d_1=0.1 mm, d_2=0.07 mm at q=6 and tg $\Delta_{xx,zz}$ = 0.5 versus frequency of the first pump wave

<u>Description of the main results obtained</u>

During this short visit, possible applications of proposed research and its extension towards Gaussian-beam scatteringwas discussed with experts of IRE NASU: Prof. A. A. Bulgakov (Dept. Solid State Radio-Physics) and IEEE Microwave Pioneer Award Winner Prof. Y. M. Kuleshov (Dept. Quasi-Optics). Possibilities of the experimental verifications of revealed effects at the millimeter and sub-millimeter waves have been also discussed.