

# Scientific report

Reference number 4603

The purpose of the visit was to continue the development of our project on Reeb dynamics for planar contact structures. In this project we are studying the existence of periodic orbits with special dynamical properties (contractible; with odd Conley-Zehnder index) for planar contact structures, something that is interesting both from the dynamical systems and the geometric perspective. One use we intend to make of this results is to prove that certain contact structures recently constructed in the work of Foulon-Hasselblat ([2]) are not planar something that is to our knowledge not obtainable by other methods.

In the visit we discussed a compactness argument that allows one to describe the breaking of a certain sequence of holomorphic curves in symplectic cobordisms and produce the special orbits we want. The main idea is to use the holomorphic openbook decompositions constructed in ([6]) and a refinement of the strategy devised in ([1]) to prove the Weinstein conjecture for planar contact manifolds. This compactness argument proves that all Reeb flows on planar contact structures must have orbits with odd Conley-Zehnder index or contractible orbits, except for some technical problems about automatic transversality of the Cauchy-Riemann operator for our broken holomorphic curves. We discussed the directions of how to attack these transversality questions during the visit and the host, Dr Chris Wendl, explained me some of his previous work ([4]) that is connected to the question: based on this we set an outline of work to solve this problem. It is important to say that previous results of Dr. Chris Wendl give very strong indication that the transversality we are trying to prove does indeed hold ([4], [5]). In fact, this is a step in a much broader project of Dr. Chris Wendl and this automatic transversality results should have other interesting applications. We also discussed the possibility of strengthening our results and of using our ideas to construct local finite energy foliations; however this is still conjectural and will be subject of future investigations of the host and the visitor.

In other direction, we studied the papers ([2],[3]) with the purpose of understanding better the dynamical obstructions to planarity that we are producing in our project and how much they overlap with currently known obstructions to planarity. After solving the problems mentioned in the second paragraph we expect to publish our results in a joint paper of the host and the visitor.

## References

- [1] C. Abbas, K. Cieliebak, H. Hofer, The Weinstein conjecture for planar contact structures in dimension three, *Comment. Math. Helv.* **80** (2005), no. 4, 771–793.
- [2] P. Foulon, B. Hasselblat, Nonalgebraic contact Anosov flows on 3-manifolds. preprint
- [3] Y. Mitsumatsu, Anosov flows and non-Stein symplectic manifolds. *Annales de l'Institut Fourier* 45, 1407–1421 (1995)
- [4] C. Wendl, Automatic transversality and orbifolds of punctured holomorphic curves in dimension four. *Comment. Math. Helv.* 85 (2) 347 - 407 (2010)
- [5] C. Wendl, Compactness for embedded pseudoholomorphic curves in 3-manifolds. *Journal of Eur. Math. Soc.* 12 (2) 313 - 342 (2010)
- [6] C. Wendl, Open book decompositions and stable Hamiltonian structures, *Expo. Math.* **28** (2010), no. 2, 187–199.