

# Scientific report on the short-visit to Helsinki in February 2012

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## 1. Purpose of the visit

The purpose of this visit is to work with Jouko Väänänen on the proposed project **Boolean-valued second-order logic**. Jouko Väänänen is one of the main figures in the research of second-order logic and bringing my knowledge of large cardinals and forcing absoluteness, we aim to solve several problems in this topic.

## 2. Description of the work carried out during the visit

I discussed with Jouko Väänänen on the proposed project and have obtained some results on the logical cardinal invariants of Boolean-valued second-order logic such as Hanf number and the least  $\kappa$  with  $\kappa$ -compactness. We compared those cardinals for Boolean-valued second-order logic with those for full second-order logic and proved that the former ones are smaller than the latter ones under some assumptions of large cardinals and  $\Omega$ -logic. Combining the results we already obtained before this visit on the complexity of the validity relations for those logics, these new results indicate that Boolean-valued second-order logic is simpler than full second-order logic.

In logic seminars in Helsinki and Bonn, I gave talks about these new results as well as the previous results and got positive response and comments from some audience.

## 3. Description of the main results obtained

We showed the following:

**Theorem 1.** Suppose there is a proper class of Woodin cardinals, a supercompact cardinal, and assume Strong  $\Omega$ -conjecture holds. Then the Hanf number for Boolean-valued second-order logic is less than the first supercompact cardinal, where Strong  $\Omega$ -conjecture is the completeness for  $\Pi_2$ -formulas with set parameters in  $\Omega$ -logic.

This contrasts the following result by Magidor:

**Theorem 2** (Magidor). If there is an extendible cardinal, then the Hanf number for full second-order logic is between the first supercompact cardinal and the first extendible cardinal.

**Corollary 3.** Suppose there is a proper class of Woodin cardinals, an extendible cardinal, and assume Strong  $\Omega$ -conjecture holds. Then the Hanf number for Boolean-valued second-order logic is strictly smaller than that of full second-order logic.

We also proved the following:

**Theorem 4.** Suppose there is a proper class of Woodin cardinals, a supercompact cardinal  $\kappa$ , and assume Strong  $\Omega$ -conjecture holds. Then  $\kappa$ -compactness holds in the Boolean-valued second-order logic with less than  $\kappa$ -many conjunctions, disjunctions, and less than  $\kappa$ -sequences of quantifiers.

This also contrasts the following result of Magidor:

**Theorem 5** (Magidor). Let  $\kappa$  be an uncountable cardinal. Then the following are equivalent:

- (a)  $\kappa$ -compactness holds in the full second-order logic with less than  $\kappa$ -many conjunctions, disjunctions, and less than  $\kappa$ -sequences of quantifiers.
- (b)  $\kappa$  is extendible.

Therefore, the least  $\kappa$  where  $\kappa$ -compactness holds in the Boolean-valued second-order infinitary logic is strictly smaller than the  $\kappa$  for full second-order infinitary logic.

4. Projected publications/articles resulting or to result from the grant.

The results obtained during this visit will be in the paper we are finishing on Boolean-valued second-order logic.