SCIENTIFIC REPORT FOR ITGP - SHORT VISIT GRANT

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1. Purpose of the visit

The purpose of my visit to CRM's research program on Geometry and Quantization of Moduli Spaces was to work on the following two ongoing questions related to my Thesis:

(P1) The question of how the picture changes when adding a real structure to the base of the principal Hitchin fibration.

(P2) The description of the moduli space of principal G-Higgs bundles through the natural spectral data that Higgs pairs carry when G is a non-compact real form.

These questions are closely related to to the research of the participants and organizers of the program, as well as the research of other mathematicians from Barcelona, making the research visit an ideal one.

Although my previous research has involved Higgs bundles over Riemann surfaces, it is interesting to consider the same setting when a real structure is added to the curve, and this constitutes my main research problem, **(P1)**. Work has been done on the Hitchin fibration for arbitrary curves. As many participants of the research program are experts in the field, the visits to CRM will allow me to start many fruitful collaborations.

One of the aims of **(P2)** is to be able to give a general method to calculate the number of connected components of the moduli space of principal G-Higgs bundles without constraints needed. The work that I have done during the last years using spectral data, already contributes to the above aim.

One should note that the two problems **(P1)** and **(P2)** are interrelated and can be studied simultaneously, hopefully leading to new interesting questions. The results that we expect to obtain should be of interest not only for researchers on Hitchin fibrations but also for people working on representation theory, mathematical physics and Langlands program. By means of the spectral data description of the moduli space of Higgs bundles, researchers would have new tools both to solve open questions and to find new interesting directions to investigate.

2. Description of the work carried out during the visit

During the visit to CRM, the work carried out in order to tackle the above problems (P1) and (P2) was done in two different ways:

2.1. **Communication of current advances.**
During the research visit, I communicated my current results in the subject (obtained in the last few months) to an audience of experts in the field as a speaker at the

Workshop on Geometry of Surface Group Representations, CRM.

The interaction with researchers in my area led to many new interesting ways of understanding my current results, opening doors for further applications of the geometric description of connected components of the moduli space of stable Higgs bundles for some non-compact real forms.

Moreover, I was also invited to give a talk about my current work at the Universitat Politecnica de Catalunya, and thus participated of the

Seminario de Geometría, Facultat de Matematiques i Estadistica

where the seminar was intended to an audience of geometers whose research was not related directly to my work. This was a challenging and fruitful experience, since by setting the talk to a less advanced level, simpler questions arose which appear to relate the current results with descriptions existing in the literature (e.g., Hitchin's papers of 1987 and 1992).

2.2. **Work on open problems.**
Further to communicating my current research, I also dedicated the visit to continue my research on open problems related to Higgs bundles for real forms. In particular, I had some interesting conversations with researchers of the trimester about **(P2)**, and studied **(P1)** with Prof. Florent Schaffhauser, who was visiting the CRM too.

3. Description of the main results obtained

The main results obtained during the visit, as well as in the days preceding and succeeding the visit, are related to Higgs bundles for the non-compact real form $Sp(2p, 2p)$. In particular, I have been able to show the following:

Theorem 1. *Each stable $Sp(2p, 2p)$-Higgs bundle (E, Φ) whose characteristic polynomial is the square of an irreducible polynomial has an associated pair (S, M) where*

(a) *$\rho : S \to \Sigma$ is a smooth $2p$-fold cover of Σ with equation*

$$\eta^{2p} + \tilde{a}_1 \eta^{2p-2} + \ldots + \tilde{a}_{p-1} \eta^2 + \tilde{a}_p = 0,$$

for η the tautological section of ρ^K and $\tilde{a}_i \in H^0(\Sigma, K^i)$, which has a natural involution $\sigma : \eta \mapsto -\eta$.

(b) *M is a stable rank 2 vector bundle on S for which $\sigma^* M \cong M$ and such that*

$$M \cong M^* \otimes \rho^* K^{2p-1}.$$

Conversely, a pair (S, M) satisfying conditions (a) and (b) defines a stable $Sp(2p, 2p)$-Higgs bundle.

This promises to be a very interesting new point of view of Higgs bundles, since for the first time the moduli space of stable rank 2 vector bundles on a curve appears related to the moduli space of stable principal Higgs bundles - previously, e.g. Hitchin 1987, line bundles on a curve were considered to study the moduli space of Higgs bundles.

4. Projected publications / articles resulting or to result from the grant (ESF must be acknowledged in publications resulting from the grantee's work in relation with the grant)

I expect to publish the work on $Sp(2p, 2p)$-Higgs bundles - addressing **(P2)**- during the next academic year, after the completion of my thesis.

Moreover, Prof. Florent Schaffhauser and I expect to continue our collaboration on Higgs bundles and real structures, addressing **(P1)**, and hence obtaining new results which all together shall be submitted for publication in the near future.