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ESF Grant, Reference Number: **4696**
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Scientific Report
Short Visit Grant No. 4696: May 7th - May 26th, 2012

The research topics in this visit were several questions in the area of Boolean algebras.

Definition (a) Let X be a compact Hausdorff space and κ be an uncountable regular cardinal. Then X is **κ-Corson**, if X is homeomorphic to a subspace Y of a Tichonov cube $[0,1]^\lambda$, such that for every $y \in Y$, $|\{\alpha \in \lambda \mid y(\alpha) \neq 0\}| < \kappa$.

(b) A Boolean algebra B is a **κ-Corson** Boolean algebra, if its Stone space $\mathrm{Ult}(B)$ is κ-Corson. □

"κ-Corson-ness" is a generalization of the notion of a "Corson compactum" which in our terminology is just an $\aleph_1$-Corson space.

We study the question whether some of known classes of Boolean algebras may have κ-Corson spaces among their members.

It turns out that κ-Corson-ness of a Boolean algebra translates nicely to the language of Boolean algebras.

Theorem Let B be a Boolean algebra. Then B is κ-Corson iff there is a subset $G \subseteq B$ such that

(i) G generates B.

(ii) For every $p \in \mathrm{Ult}(B)$, $|p \cap G| < \kappa$.

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We proved the following theorem.

Theorem A Let κ be an infinite successor cardinal. If B is a Boolean algebra which is a poset algebra, and $|B| = \kappa$, then B is **not** κ-Corson. □

Remark The class interval algebras is a subclass of the class of poset algebras. So Theorem A applies to the class of interval algebras.

Theorem A together with some additional results on κ-Corson Boolean algebras have been already written up, and will be submitted soon.

I also gave two lectures in the Balcar and Simon Seminar:

(1) On the existence of a subalgebra of $B(\mathbb{R})$ which is not an interval algebra.

(2) On the classification of scattered posets with the finite anti chain condition.

Projected publications / articles resulting or to result from the grant

[BK] R. Bonnet and W. Kubiś: Ultrafilter selection properties of Boolean algebras (preprint), 22 pp.

R. Bonnet