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Final Report

The purpose of my visit to Irene Amado at SISSA was to work on our ongoing collaboration about the possibility of Hawking–Page phase transitions in systems enjoying anisotropic scale invariance. This work is a follow up of our previous publication [1], where we found, numerically, the first (and so far unique) black hole solutions with Lifshitz asymptotics in string theory. These geometries are conjectured to be dual to systems invariant under the anisotropic scale transformation

$$t \to \lambda^z t\,, \qquad\qquad x \to \lambda x \tag{1}$$

where t and x are temporal and spatial coordinates respectively [2]. Placing a black hole in the bulk provides temperature to the setup. The goal is to holographically model the Lifshitz field theory that is supposed to arise as the long wavelength (continuous) limit of lattice models of strongly correlated electrons, and thus these geometries are of interest in condensed matter applications.

In [1] we also conjectured, based on the appearance of a minimum temperature for the existence of the black holes, that the system should undergo a phase transition if we continue decreasing the temperature from that point. In analogy with the well known case of asymptotically AdS spaces, the transition is expected to be similar to the Hawking–Page one. The interpretation in the dual theory is nevertheless much less clear.

The first thing to do is to figure out what would be the endpoint of the transition. In the standard AdS/CFT case the suitable geometry describing the low temperature state is an AdS soliton with the temporal coordinate periodically identified. This solution is often called “thermal AdS”. Then, one would need to find a soliton with the appropriate Lifshitz asypmtotics. These type of solitons were known to exist in phenomenological (botton-up) studies of Lifshitz geometries, but a similar solution in string theory was lacking. We have by now found the “thermal Lifshitz” states in the same model in which we constructed the black holes, so we have a candidate for the low temperature phase of the system. It turns out that the space of parameters of our solitons has different properties than the phenomenological ones, which is interesting by itself.

The conclusive evidence for the presence of the phase transition is the behaviour of the free energy. This is computed by evaluating the on-shell (Euclidean) action for a given solution. The solution with the lowest free energy at any temperature would be the dominating phase at that temperature. Unfortunately, the on-shell action is divergent as it stands. One is then forced to renormalize it by computing the necessary counterterms, which are local operators on the bounday, constructed with the metric and the matter fields. This procedure is well known in the case of asymptotically AdS spaces. In the case of asymptotically Lifshitz spaces, the development of the analogous formalism is very recent [3]. The particular form of the counterterms is matter dependent, so we had to compute them for the model we are studying, and this task was achieved during my visit at SISSA.

In the AdS case there is a special counterterm that kills logarithmic divergences, and that corresponds in the dual gauge theory to the Weyl anomaly [4], that is, the breaking of scale invariance at the quantum level. Very recently, it has been observed that anisotropic scale invariance of the type (1) can also be anomalous [5, 6]. Indeed, we have found such an anisotropic anomaly in our system.

One of the observations of [5, 6] is that the anisotropic anomaly, in the minimal models that support Lifshitz metrics, contains just temporal derivatives of the boundary metric. We have shown that in more general models, like the one we have at hands, this is no longer true, and the square of the boundary curvature enters in the anomaly, so this property is dependent on the matter content.

Perhaps more interestingly is the observation that we need to kill not only logarithmic divergences but also terms that diverge as logarithm square. This seems to be in contrast with the asymptotically AdS case, and is due to additional powers of the radial coordinate in the integration measure. The dual interpretation of these completely new term is still unclear to us.

Once we have a finite on-shell action, it is very simple to compute from it the relevant thermodynamic quantities, such as the pressure and the internal energy, by taking the appropriate elemets of the boundary energy-momentum tensor. We are currently involved in computing such quantities both for the Lifshitz black hole and the solitons. Unfortunately the numerics are extremely sensible so the progress is slow.

In the future, it would be very interesting to investigate the dual interpretation of the anisotropic phase transition. In the usual AdS example, it is well known that the transition between AdS black holes and thermal AdS corresponds in the gauge dual to a confinement-deconfinement transition. Since the holographic dictionary is still underdeveloped for asymptotically Lifshitz geometries, the nature of the transition is obscure.

On the other hand, we need to interpret the coefficient of the logarithm squared divergent term. As we mentioned, the logarithmically divergent term is associated to the anisotropic Weyl anomaly. We may be finding a new type of anomaly or an exotic correction to the usual breaking of the scale invariance.

We hope to finish this calculations in the next weeks and have a definitive answer to the question of the phase transition.

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