FINAL REPORT FOR THE INFTY SHORT VISIT GRANT 4862

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Young Set Theory Meeting 2012

Report from the visit.

During my stay in Luminy, I had an opprtunity to work with Michal Doucha, who is a Ph.D. student of Jindra Zapletal, and discuss the problems of canonization of Borel equivalence relations on nondominating subsets of the Baire space and on Silver cubes.

A class of Borel equivalence relations has *total canonization on nondominating subsets of the Baire space* (or *Silver cubes*) if for every Borel equivalence relation E in that class there is a Borel nondominating set C (or a Silver cube, resp.) such that E restricted to C is one of the two trival equivalence relations.

While it is not possible to canonize all equivalence relations neither on nondominating sets nor on Silver cubes, one can hope for total canonization for some restricted classes of equivalence relations.

One important example here is the measure equivalence relation, which is the equivalence relation on the space of all measures on the Cantor set defined so that two measures are equivalent if they are absolutely continuous with respect to each other.

The measure equivalence relation is a Borel equivalence relation that is quite complicated with respect to the Borel reducibility order and the class of Borel equivalence relation that are reducible to the measure equivalence relation is quite rich (e.g. it contains the relation $=^+$).

During my stay in Luminy, I was working with Michal Doucha on the following conjecture:

Conjecture. *The class of all Borel equivalence relation reducible to the measure equivalence relation admits total canonization of Silver cubes.*

Our discussion concentrated on the analysis of Michal Doucha's proof of canonization of equivalence relations reducible to the measure equivalence relation on nondominating subsets of the Baire space. The proof is quite challenging and I suspect that it can serve as a basis to the attack on the above conjecture.

This work was connected with an ongoing project with Jindra Zapletal and Vladimir Kanovei on the general problem of canonization of Borel equivalence relations. With Michal Doucha, we came to a good understanding of the problem and got many promising ideas that will stimulate the future work.

I also had an opportunity to participate in the conference Young Set Theory Workshop 2012 and listen to the four tutorials:

- Ilijas Farah, Some applications of set theory to operator algebras,
- Alain Louveau, Introduction to Effective Descriptive Set Theory,
- Itay Neeman, Forcing iterations with finite side conditions,
- Stevo Todorcevic, Walks on ordinals and their characteristics.

I benefited a lot from the tutorial by Ilijas Farah, who presented a self-contained survey of the basics of C^*-algebras. The tutorial was accessible to a set-theorist and put the emphasis on the currect applications of combinatorial and descriptive set theory to C^*-algebras. The connections between set theory and functional analysis seem to be one of the most promising directions for young set-theorists and the tutorial gave me a good understanding of the problems that can be attacked. A good example is the problem of classification of compact metric spaces up to homeomorphism, which turns out to have an interesting connection with the classification of C^*-algebras.