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Reporting requirements
European Science Foundation - Short Visit Grants
Contact and Symplectic Topology

To Whom it May Concern:

Purpose of the visit. I visited Prof. Dr. Kai Cieliebak in the Mathematics Department at the Ludwig-Maximillans Universität München, Germany from June 28-July 6, 2012 to work on the well-definedness of cylindrical contact homology. This is an extremely useful invariant when properly defined. It can be used to answer many useful questions regarding qualitative behavior of Reeb vector fields and uniqueness of certain structures in the realm of contact topology and geometry. Prof. Dr. Kai Cieliebak and I worked further on establishing when cylindrical contact homology can and cannot be properly defined.

Description of the work and main results. Symplectic field theory has been around for more than a decade but significant analytic obstacles remain and very few rigorous proofs have appeared in the literature. It was sketched by Eliashberg, Givental, and Hofer [EGH00] in 2000. As commented in the introduction of their paper, it contained practically no proofs and despite the fact that more than a decade has passed, unfortunately even today complete arguments have yet to be presented. This original article was meant as a propoganda paper of what contact homology should be if all the analytic difficulties could be resolved.

Cylindrical contact homology is one of the "simplest" invariants coming from this framework, but in most cases one is unable to conclude that what one has computed is in fact a homology. For many years the issues in understanding compactness and transversality were not discussed in conjunction with cylindrical contact homology. We discovered that the previously perpetuated assumptions on the Conley-Zehnder indices of contractible closed Reeb orbits do not ensure $d^2 = 0$ due to the existence of multiply covered cylinders and their branched covers. We determined what stronger conditions are necessary on the growth rates of the indices of simple contractible orbits to obtain a well-defined homological theory.

Prof. Dr. Kai Cieliebak and I looked into the issues in concrete examples including, S^{2n+1}, the ellipsoid, and $L(n+1, n)$. We discussed what contact forms yield can and cannot be utilized in computations. I discussed my work in progress or prequantization spaces and certain S^1 bundles over nicely behaved symplectic orbifolds that satisfies these new requirements on the growth of the Conley-Zehnder indices of closed iterated Reeb orbits. We determined in several instances when this homology theory is well-defined and what transversality issues remain to be resolved before a more general setting can hold.

We also investigated when a well-defined integral grading of Conley-Zehnder indices exists for non-contractible Reeb orbits appearing in the chain groups associated to cylindrical contact homology. This is a construction coming from symplectic homology. I am currently writing up the details of this.

I gave a 2 hour seminar talk detailing some of these findings in the Symplectic Geometry Seminar at Ludwig-Maximillans Universität München on July 3, 2012.

Future collaboration with host. Prof. Dr. Kai Cieliebak and I will continue to be in contact as I finish my disseration this fall at the University of Wisconsin - Madison in the United States. My thesis will provide an in-depth explanation of all the issues involved in defining cylindrical contact homology and providing concrete well-defined examples. I have greatly benefitted from our discussions this summer and hope that my thesis will provide others with insights into the intricate and delicate nature of defining homology theories via pseudoholomorphic curves.

In addition, we discussed issues of uniqueness regarding symplectic fillings of links of simple singularities and lens spaces $L(p, q)$. We plan to further investigate these new directions after work on establishing the well-definedness of cylindrical contact homology is completed.

Projected publications. We project that there will be at least 1-2 publications resulting from this grant. I am currently writing up a paper detailing what is mentioned in this report as well as my own methods for computing cylindrical contact homology prequantization spaces. An Acknowledgement of the support provided by the ESF will of course be indicated in any publications.

Sincerely yours,

Joanna Nelson

References

[EGH00] Y. Eliashberg, A. Givental, and H. Hofer, Introduction to Symplectic Field Theory, *Geom. Funct. Anal.*, Special Volume, Part II: 560-673, 2000.