Scientific report - Grzegorz Jabłoński
Visit in IST Austria (Host: prof. Herbert Edelsbrunner).

1. Purpose of the visit:
   Establish collaboration with prof. Herbert Edelsbrunner on persistent homology of maps. Prepare an algorithm to study the parameters of maps given by point cloud data.
2. Description of the work carried out during the visit:
   1. Aim: Preparation of algorithm to compute persistent homology of point cloud data (representing unknown map). Let assume A are points in topological space X. Then one can obtain pairs (x,y) from A x A. Such points represents an unknown function f. Every pair consist of the argument and the value of function s.t. f(x) = y. Such point data may be obtained from experiments in physics, biology or engineering.
      The standard methods focus on topological aspects of points, not the given function. Thus we want to introduce methods to investigate such situation.
   2. Used method: We proceed as follow: we split the f function into two projections from graph of the function (graph of function is set of points consisting of argument and value of the function). First projection (we call it later p) is a projection onto the argument. The second projection is onto the value of the function. In such situation we have f=q p^-1. Thus one can use the generalized eigenvalue problem and study the problem of subspace spanned by the kernel of p- lambda*q, where lambda is some value given by the user. Common method, which is also used in our algorithm, to get the homology group from the set of points is to use sequence of Vietoris Rips complexes. In the first step we compute the matrix representation of the projections p and q in the homology group of the Vietoris Rips complexes. This part is based on algorithm presented in „ Computing linking numbers of a filtration.” Homology, Homotopy and Applications 5 (2003), (authors:H. Edelsbrunner, A. Zomorodian.). In the second step we compute the desired kernel. It could be done straightforward using methods from the book „Computational Homology” by Tomasz Kaczyński, Konstantin Michael Mischaikow and Marian Mrozek. In the last step using algorithm based on „Zigzag Persistence” (Gunnar Carlsson, Vin de Silva, 2008) we study the persistence of the eigenspaces in such setting.
3. Description of the main results obtained:
   We prepared the foundations of the algorithm, also some initial implementations. From the very first experiments we could say, that algorithm gives us expected information. So one can then classify the function based on the eigenvalues. If two functions could be represented by the same eigenvalue, we could assume they are homotopy equivalent. This result could be used in the study of the dynamical systems and times series, where one is interested in classifying homotopic functions (whose properties are similar).
   However the most important fact is that the algorithm could be build on top of the standard algorithm used to compute persistent homology. Such algorithm should be then stable and fast in average situation.
4. Future collaboration with host institution
   Planned future collaboration with prof. Herbert Edelsbrunner. In case of additional funding longer visit is possible.
5. Projected publications / articles resulting or to result from the grant
   In preparation: „Persistent homology of a map”, Herbert Edelsbrunner, Marian Mrozek, Grzegorz Jabłoński
6. Other comments:
   Another scientific activities during my stay:
   1. Attendance to Computational Geometry and Topology lectures
   2. Attendance to Scientific Presentation and Conduct lectures