ESF Research Networking Programme NEWFOCUS
Final scientific report

Project Title

Complex Conical Beams in Reflector System Analysis

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Exchange Period

22/08/2012 – 05/09/2012 (15 days)

1. PURPOSE OF THE VISIT

Recently, a new propagation model built around special wave objects called complex conical beams has been introduced, and a method for analysing reflector antenna systems based on a combination of that propagation model and Physical Optics has been developed. The new hybrid method was shown to be a well-suited approach for high-frequency problems, being essentially significantly faster than pure Physical Optics and comparably accurate.

At the current stage, the project is concerned with extending the capabilities of the developed analysis method. It has been found, however, that for many of the possible applications, the Physical Optics part is the bottleneck of the analysis, limiting the achievable gain in speed. The goal of the visit was therefore to discuss the abovementioned and other issues related to the application of complex beams in reflector and lens antenna analysis with scientists of IRE NASU who work in the same area of computational electromagnetics and jointly seek for improved formulations, which will allow to incorporate CCB analysis into other analysis workflows and extend the range of its possible applications.

2. DESCRIPTION OF THE WORK CARRIED OUT DURING THE VISIT

The majority of the scientific short visit consisted of visits to the Institute of Radio-Physics and Electronics NASU and involved presentations and discussions with researchers from the home institution, as summarized in the table below. Furthermore, from 28th August thru 31st August, I also participated in the MMET'12 conference (Mathematical Methods in

Electromagnetic Theory), which is organised by scientists from IRE NASU and is the leading forum for exchanging know-how and ideas in the field of computational electromagnetics in the former USSR area, gathering almost 150 scientists from Ukraine, Russia and the rest of the world.

Table 1. Program of the short visit

DATE	VENUE	PROGRAM / TOPIC	PARTICIPANTS
Thursday, 23/08/2012	IRE NASU	visit to the Institute and the laboratory, general discussion	S. Skokic, A. Nosich, M. Balaban, V. Byelobrov
Friday 24/08/2012	national holiday		
Monday 27/08/2012	IRE NASU	CCB-based analysis, presentation & discussion	S. Skokic, M. Balaban, A. Nosich
Tuesday 28/08/2012	National Academy of Municipal Economy	MMET'12 Conference	
Wednesday 29/08/2012	National Academy of Municipal Economy	MMET'12 Conference	
Thursday 30/08/2012	National Academy of Municipal Economy	MMET'12 Conference	
Friday 31/08/2012	conference social event		
Monday 03/09/2012	IRE NASU	thin disc scattering analysis, presentation & discussion	M. Balaban, S. Skokic
Tuesday 04/09/2012	IRE NASU	concept of complex plane boundary conditions, discussion; discussion about future collaboration	S. Skokic, M. Balaban, I. Sukharevsky

2.1. CCB-Based Analysis of Reflector Systems (S. Skokic)

The basic formulation of CCB's (type A) starts with the scalar radiation integral in spectral domain:

$$I'(x,y,z) = \frac{1}{8\pi^2 j} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{F}\left(k_x, k_y\right) \frac{e^{-j\left(k_x x + k_y y\right) - jz\sqrt{k^2 - k_x^2 - k_y^2}}}{\sqrt{k^2 - k_x^2 - k_y^2}} dk_x dk_y \qquad (1)$$

where $\tilde{F}\left(k_x, k_y\right)$ denotes the spectrum of the aperture field, and $I'(x,y,z)$ is the radiated spatial domain potential away from the aperture. By switching to cylindrical coordinates, introducing the Fourier series in the spectral azimuthal variable α and subsequently applying the GPOF approximation on the obtained Fourier coefficients, the double integral from Eq. (1) is replaced by this double summation

$$I(\rho,\phi,z) = \sum_{n} \sum_{m=1}^{M} a_{mn} W_n(\rho,\phi,z + jb_{mn}) \qquad (2)$$

where W_n denotes the *n*-th order complex conical beam, and is defined by

$$W_n\left(\rho,\phi,\tilde{z}\right) = e^{jn\phi} \int_{0}^{\infty} \frac{e^{-j\tilde{z}\sqrt{k^2 - k_\rho^2}}}{\sqrt{k^2 - k_\rho^2}} \mathrm{J}_n\left(\rho k_\rho\right) k_\rho dk_\rho \, . \qquad (3)$$

The integral in (3) possesses an analytical recursive solution which can be calculated with help of the so-called Gradshteyn integral

$$\frac{\partial I_n^{(g)}}{\partial \rho} = (-j)^{n+1}\frac{k\pi}{8}\frac{\rho}{r}\Big[\mathrm{J}_{n/2+1}(d^-)\,\mathrm{H}^{(2)}_{n/2}(d^+) - \mathrm{J}_{n/2-1}(d^-)\,\mathrm{H}^{(2)}_{n/2}(d^+) + \mathrm{J}_{n/2}(d^-)\,\mathrm{H}^{(2)}_{n/2-1}(d^+) - \mathrm{J}_{n/2}(d^-)\,\mathrm{H}^{(2)}_{n/2+1}(d^+)\Big], \tag{4}$$

with $d^- = -\frac{1}{2}k(r-z)$ and $d^+ = \frac{1}{2}k(r+z)$.

For PEC reflector system analysis, this propagation model was linked to the Physical Optics (PO) method. More recently, in research partially supported by NewFocus programme, the analysis of frequency-selective reflector systems has been addressed. In this case, however, the PO analysis is not sufficiently accurate, and the CCB model was complemented by Spectral-Domain MoM analysis, which was further enhanced in order to render it applicable to the considered problem class. In both cases, the CCB analysis serves to speed up the propagation part of the analysis, while reflection and/or scattering must still be taken into account by other methods. In theory, CCB analysis can be augmented or complemented with any sufficiently fast and reasonably accurate method. The range of applicability of various analyses is illustrated in Fig. 1.

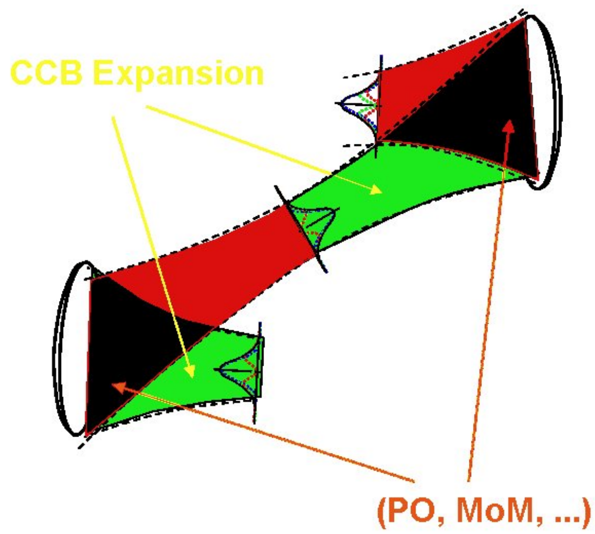

Figure 1. CCB-based reflector system analysis – decomposition

2.2. Thin Disc Scattering (M. Balaban)

Thus far, only perfectly electrically conducting bodies have been considered in CCB-based analyses. The analysis of scattering from dielectric bodies presents an even greater computational challenge. Although developed primarily for the long-wave analysis of thin dielectric discs (i.e. where disc radius is less than the wavelength), the method for the analysis developed by M. Balaban *et al.* of IRE NASU, Ukraine, is more exact than most other models and might be a possible starting point for the extension of the CCB analysis method towards the analysis of dielectric bodies. The method used by Balaban resorts to Neumann series expansion solution of the Fredholm second kind integral equation (IE) to obtain the long-wave asymptotics of the unknown functions, which are images of the electric and magnetic currents and the average values of scattered field components normal to the disk.

3. DESCRIPTION OF THE MAIN RESULTS OBTAINED

The most recent results obtained via Complex Conical Beam analysis are shown in Fig. 2, where a dual-reflector system composed of an ellipsoidal subreflector and a paraboloidal main reflector has been considered. The size of the main reflector is over 50λ in diameter. The reference solution has been computed in the industry-standard EM simulator GRASP (TM). Results show excellent agreement .

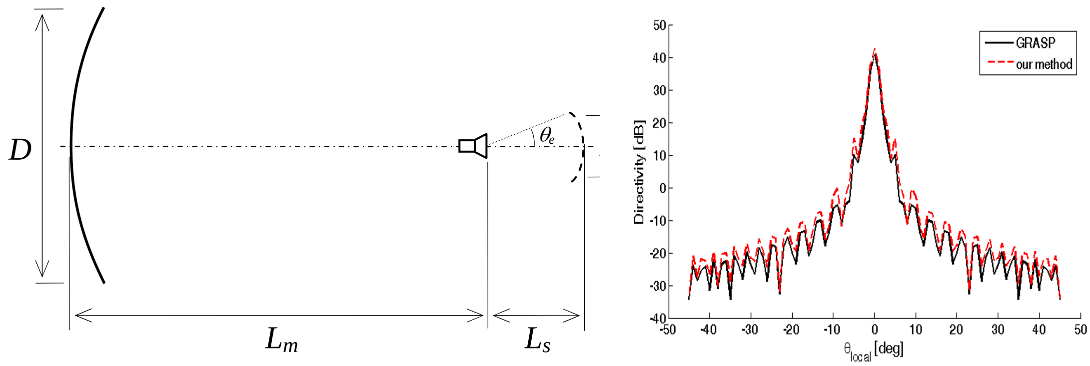

Figure 2. Gregorian dual-reflector frequency-selective antenna system and its radiation pattern calculated via SD-MoM-CCB hybrid analysis method (compared to GRASP).

An overview of CCB analysis, including the test case shown above, was presented at the MMET'12 conference (Mathematical Methods in Electromagnetic Theory), organised by IRE NASU scientists and hosting almost 150 scientists from Ukraine, Russia and worldwide. The work was very well received and was awarded the second prize in the young scientists competition. In the paper and the oral presentation, NewFocus support was duly acknowledged. The paper and a scanned copy of the award are included in the Annex of this report.

4. FUTURE COLLABORATION WITH HOST INSTITUTION

On the last day of the visit, possibilities of starting a long-term collaboration between IRE NASU and University of Zagreb were considered, within but not limited to the scope of the NewFocus network programme. The strong bacground in applied mathematics of the research group from IRE NASU would complement well the know-how and experience in modeling and design of the Department of Wireless Communications of the University of Zagreb and could be beneficial to both institutions. A short scientific visit to Zagreb in winter 2012 or spring 2013 has been proposed to young scientists of IRE NASU, including M. Balaban, V. Byelobrov and I. Sukharevsky. The realization of that idea will, however, depend on the availability of funds.

5. PROJECTED PUBLICATIONS / ARTICLES RESULTING OR TO RESULT FROM THE GRANT

Due to the scope of this short visit, no conference or journal papers will be published as a direct result of this grant. However, one more paper covering CCB analysis is expected in 2013. Therein, ESF NewFocus support will be duly acknowledged.

ANNEX 1:

Scientific paper published at the MMET*12 conference, Kharkov, 28-30 August 2012

Authors: S. Skokic, Z. Sipus, S. Maci, M. Bosiljevac and M. Casaletti
Title: Application of Complex Conical Beams in Reflector System Analysis

APPLICATION OF COMPLEX CONICAL BEAMS IN REFLECTOR SYSTEM ANALYSIS

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Abstract – **This paper presents an overview of complex conical beams and techniques developed for their application in the analysis of reflectors and reflector antenna systems, realised either as full PEC surfaces or quasi-periodic, frequency selective surfaces. The various techniques link complex conical beams with Physical Optics, Spectral-Domain method and the Uniform Theory of Diffraction. The method is tested on several examples, including a frequency-selective dual-reflector system of the Gregorian type, designed for the operating frequency of 25 GHz.**

I. INTRODUCTION

The work on beam formulations for the analysis of reflector antenna systems has been an ongoing research topic for years, because the electrical size of analysed objects is often too large for the application of classical numerical methods such as Moment Method od Finite Difference Time-Domain method. For high-frequency problems, a better-suited approach is to express the radiated field in terms of a relatively low number of wave objects of higher complexity and treat their propagation and reflection directly. In this way, the number of unknowns in the system is reduced. The aim is to develop a modular process, whereby the total reflected field from one reflector can be re-expanded into a new sum of wave objects of the same kind, and the whole procedure can be repeated in the same way for all subsequent reflectors.

The most important of our research in this area are the complex conical beams (CCB) which serve to accelerate the computation of propagation of waves between analysed elements. This is achieved using Generalized Pencil of Functions method (GPOF) and Fourier series expansion, which reduce the radiation integral to a double summation in which each element represents one *n*-th order wave object (i.e. beam). The use of GPOF method ensures a low number of elements in the sum, due to its adaptive nature.

In subsequent research, this propagation model was included in the Physical Optics analysis of reflector systems, and the new model was successfully verified for the case of one and two ellipsoidal reflectors. At the current stage, the project is concerned with extending the capabilities of the developed analysis method and incorporating CCB analysis into other analysis workflows, so as to extend also the range of possible applications of the developed analysis model.

II. FORMULATION

The basic formulation of Complex Conical Beams starts with the scalar radiation integral in spectral domain:

$$I'(x,y,z) = \frac{1}{8\pi^2 j}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{F}\left(k_x,k_y\right)\frac{e^{-j\left(k_x x+k_y y\right)-jz\sqrt{k^2-k_x^2-k_y^2}}}{\sqrt{k^2-k_x^2-k_y^2}}dk_x dk_y \tag{1}$$

where $\tilde{F}\left(k_x,k_y\right)$ denotes the spectrum of the aperture field, and $I'(x,y,z)$ is the radiated spatial domain potential away from the aperture. By switching to cylindrical coordinates, introducing the Fourier series in the spectral azimuthal variable α and subsequently applying the GPOF approximation on the obtained Fourier coefficients, the double integral from Eq. (1) is replaced by this double summation

$$I(\rho,\phi,z) = \sum_n \sum_{m=1}^{M} a_{mn} W_n(\rho,\phi,z+jb_{mn}) \tag{2}$$

where W_n denotes the *n*-th order complex conical beam, and is defined by

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$$W_n\left(\rho,\phi,\tilde{z}\right)=e^{jn\phi}\int_0^\infty \frac{e^{-j\tilde{z}\sqrt{k^2-k_\rho^2}}}{\sqrt{k^2-k_\rho^2}}\mathrm{J}_n\left(\rho k_\rho\right)k_\rho\,dk_\rho \tag{3}$$

The integral in (3) possesses an analytical solution, albeit a recursive one. It has been demonstrated in [5] that a (n+1)-th order CCB can be computed from (n-1)-th CCB as

$$W_{n+1}=e^{j2\phi}W_{n-1}-2je^{j(n+1)\phi}\frac{\partial}{\partial\rho}I_n^{(g)} \tag{4}$$

where $\frac{\partial}{\partial\rho}I_n^{(g)}$ is the ρ-derivative of the so-called Gradshteyn integral [6]

$$\begin{aligned}\frac{\partial I_n^{(g)}}{\partial\rho}=\left(-j\right)^{n+1}\frac{k\pi}{8}\frac{\rho}{r}\Big[&\mathrm{J}_{n/2+1}\left(d^-\right)\mathrm{H}_{n/2}^{(2)}\left(d^+\right)-\mathrm{J}_{n/2-1}\left(d^-\right)\mathrm{H}_{n/2}^{(2)}\left(d^+\right)\\&+\mathrm{J}_{n/2}\left(d^-\right)\mathrm{H}_{n/2-1}^{(2)}\left(d^+\right)-\mathrm{J}_{n/2}\left(d^-\right)\mathrm{H}_{n/2+1}^{(2)}\left(d^+\right)\Big],\end{aligned} \tag{5}$$

with $d^-=-\frac{1}{2}k(r-z)$ and $d^+=\frac{1}{2}k(r+z)$.

III. Application To Reflector System Analysis

For the analysis of reflector systems, this propagation model is linked with Physical Optics analysis [4], [5], however the beams first have to be vectorised to be able to represent actual electromagnetic fields [2]. This is achieved by interpreting the scalar radiation integral in (1) as components of the vector potential [5], from which the radiated field can be obtained via differentiation

$$\mathbf{E}=\hat{x}\left(\frac{1}{\varepsilon}\frac{\partial F_y}{\partial z}\right)-\hat{y}\left(\frac{1}{\varepsilon}\frac{\partial F_x}{\partial z}\right)+\hat{z}\left(\frac{1}{\varepsilon}\left(\frac{\partial F_x}{\partial y}-\frac{\partial F_y}{\partial x}\right)\right). \tag{6}$$

The analysis of a beam waveguide using wave-objects consists of the following basic steps, which must be done for each reflector, except for the first and the last one (the first reflector may not need step 1, while the analysis of the last reflector obviously stops at point 3):

1) calculate field spectrum in the input plane
2) expand the incident field into complex conical beams
3) compute equivalent currents on the reflector surface using complex conical beams
4) calculate reflected field via Physical Optics or some other method (eg. SD-UTD method for FSS)

An important aspect of the system analysis is the efficient calculation of the electric field spectrum in the input/output plane. Typically, a 2D Fast Fourier Transform is employed for that purpose. However, the CCB expansion requires spectrum samples on a polar grid. Two better alternatives to the classical FFT were found: 1) the recently introduced Pseudo-Polar FFT algorithm (PPFFT) [6], and 2) the use of inverse Near-Field to Far-Field transformation (NF-FF) [7],[8].

For the first test case, we chose a scenario of two ellipsoidal reflectors, which represents a typical section of a beam waveguide. The reason for this is easily seen from classical optics. If the source is placed in the first focal point of an ellipsoidal reflector, then, upon reflection, the reflected beam will focus in the plane containing the second focal point of that reflector, which is optimal in the sense of keeping the beam tight as it propagates through a sequence of reflectors.

The geometry of the analysed two-reflector system is shown in Fig. 1. The frequency is 3 GHz, i.e. λ = 0.1 m, and the semi-axes of both ellipsoids are a = 5 m = 50λ, b = c = 4 m = 40λ. The first reflector is illuminated from its geometrical focus F1 by a complex Huygens source with a beam taper of A = 10 dB and a beam taper angle of θ = 6°, which corresponds to the beam waist $w_{0,inc}$ = 0.326 m = 3.26λ. The incident beam is launched towards the apex of the first ellipsoid (point (0, 0, 4 m) on the z-axis).

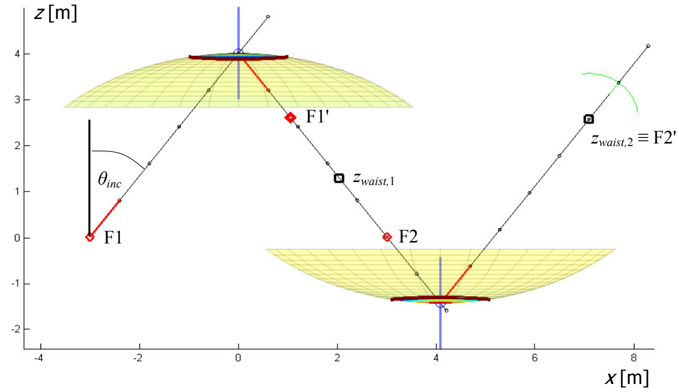

Fig. 1. Side view of a system of two ellipsoidal reflectors (circular rim). Green lines denote far-field observation points.

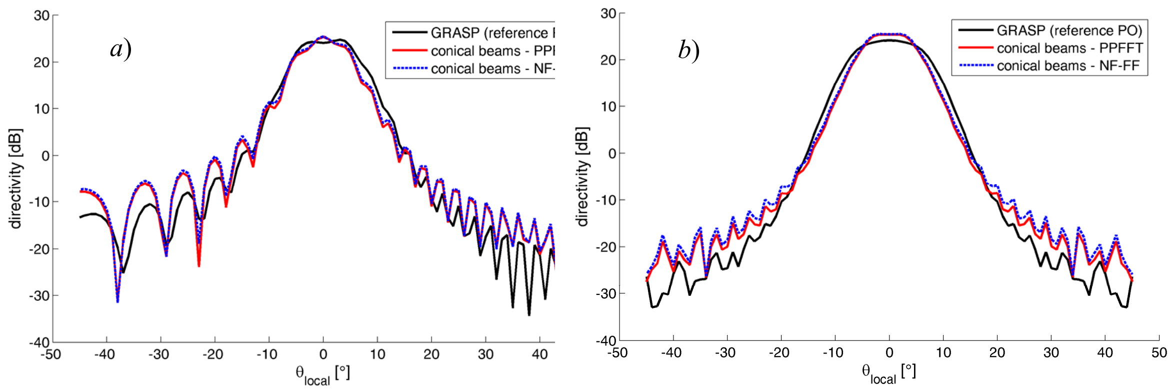

Fig. 2. Radiation pattern (directivity) of the output beam after two reflections; *a*) $\phi_{local} = 0°$, *b*) $\phi_{local} = 90°$.

By linking the conical beams with hybrid Spectral-domain UTD analysis approach, the capabilities of the CCB-based analysis have been extended to include also curved frequency-selective surfaces. The hybrid SD-UTD method combines the advantages of the spectral domain approach, such as the possibility of analysing multilayered structures, and the UTD method, which allows for the analysis of generally shaped convex surfaces. In the spectral domain approach, a three-dimensional problem is transformed into a spectrum of one-dimensional problems by performing a two-dimensional Fourier transform in the coordinates for which the structure is homogeneous. As a result, only the electromagnetic field variation in the direction normal to the surface is unknown. This allows for the rigorous analysis of canonical multilayered structures. To be able to analyse electrically large reflectors, the spectral domain approach was accelerated using various techniques [9].

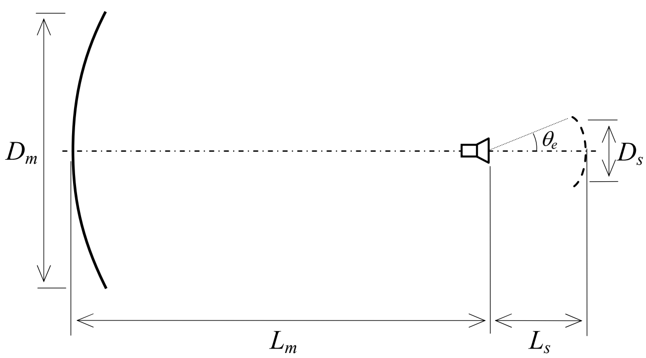

Fig. 3. Gregorian dual-reflector frequency-selective antenna system considered in the analysis; the subreflector is ellipsoidal, while the main reflector is paraboloidal. $D_m = 53$ cm $\approx 44\lambda$, $L_m = 28.22$ cm, $D_s = 5.3$ cm, $L_s = 4.16$ cm. The focal lengths of the reflectors are $F_m = 28.99$ cm and $F_s = 3.8$ cm, respectively. The taper angle of the feed is $\theta_e = 40.7°$.

To verify the obtained method, we analysed a dual-reflector system of the Gregorian type, composed of an ellipsoidal frequency-selective subreflector and a paraboloidal main reflector, as shown in Fig. 3. Both reflectors were rotationally symmetric, and were designed following the recommendations given in [10] for the working frequency of 25 GHz, starting from the dimensions of the frequency selective grid which formed the subreflector. The feed was modeled as a $\cos^n(\theta)$ source, where the exponent n was chosen such as to achieve attenuation of 10 dB at the edge of the subreflector (corresponding to the taper angle θ_e, as shown in Fig. 3). The obtained results and their comparison to reference results calculated with GRASP (TM) are shown in Fig. 4.

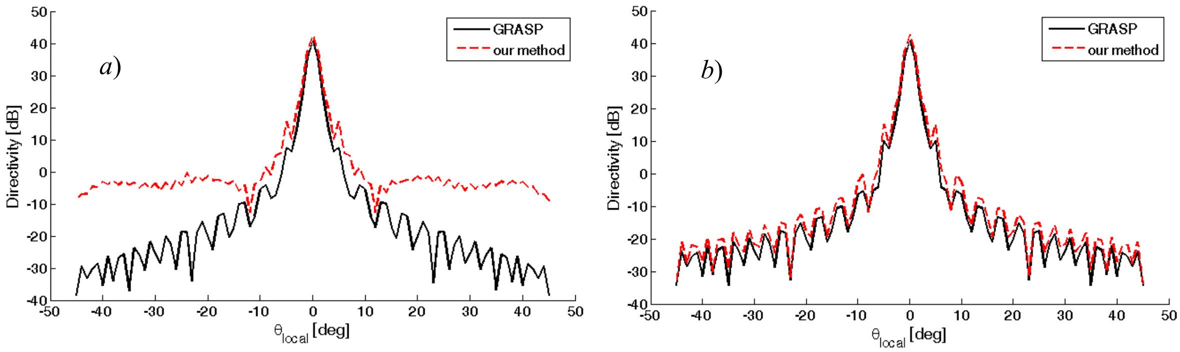

Fig. 3. Radiation pattern of the designed reflector system at 25 GHz: a) E-plane; b) H-plane

IV. Conclusion

In this paper, we have presented an overview of the developed techniques for the analysis of reflector antenna systems based around complex conical beams. The developed propagation model has been linked to other analysis methods such as Physical Optics and the Spectral Domain approach in order to extend the range of its potential applications. For validation purposes, different dual-reflector systems were designed and the obtained results have been successfully compared to GRASP simulations, proving the potential of conical beams.

Acknowledgement

This research was in part supported by European Science Foundation project ESF RNP NewFocus: “New Frontiers in mm/sub-mm waves integrated dielectric focusing systems”.

References

[1.] M. Casaletti, S. Skokic, S.Maci and S. Sørensen “Beam Expansion in Multi-reflector Quasi-Optical Systems”, Proc. 4th European Conference on Antennas and Propagation (EUCAP 2010), Barcelona, Spain, 2010.
[2.] S. Skokic, M. Casaletti, S.Maci and S. Sørensen “Complex Conical Beam Expansion for the Analysis of Beam Waveguides”, Proc. 3rd European Conference on Antennas and Propagation (EUCAP 2009), Berlin, Germany, 2009.
[3.] T. K. Sarkar and O. Pereira, “Using the Matrix Pencil Method to Estimate the Parameters of a Sum of Complex Exponentials”, IEEE Antennas and Propagation Magazine, vol. 37, no. 1, Feb. 1995, pp. 48-55.
[4.] T. Bondo and S. Sørensen “Physical Optics Analysis of Beam Waveguides Using Auxiliary Planes”, IEEE Trans. Antennas Propagat., vol. 53, No. 3, Mar. 2005, pp. 1062-1068
[5.] S. Skokic, “Analysis of Reflector Antenna Systems by Means of New Conical Wave Objects”, Ph.D. Thesis, University of Zagreb, Croatia, 2010.
[6.] A. Averbuch, R. R. Coifman, D. L. Donoho, M. Elad and M. Israeli, "Fast and Accurate Polar Fourier Transform", *Appl. Comput. Harmon. Anal.*, vol. 21, p.p.145–167, 2006.
[7.] C. A. Balanis, *Antenna Theory – Analysis and Design*, 2nd edition, John Wiley and Sons, NY, USA, 1997.
[8.] J. J. H. Wang, "An Examination of the Theory and Practices of Planar Near-Field Measurement", *IEEE Trans. Antennas & Propagation*, vol. 36, no. 6, pp.746-753, 1988.
[9.] Z. Sipus, P.-S. Kildal, R. Leijon, and M. Johansson, ”An algorithm for calculating Green’s functions for planar, circular cylindrical and spherical multilayer substrates,” *Applied Computational Electromagnetics Society Journal*, Vol. 13, No. 3., pp. 243-254, Nov. 1998.
[10.] C. Granet, “Designing Axially Symmetric Cassegrain or Gregorian Dual-Reflector Antennas from Combinations of Prescribed Geometric Parameters,” IEEE Antennas and Propagation Magazine, Vol. 40, No. 2, pp. 76-82, Apr. 1998.