Scientific report - Preservation of spectral and probabilistic properties under the approximation of Dirichlet forms

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Time and place of visit.

The visit in Japan from September 23rd until October 11th 2012 included stays at the institutes Okayama University (September 23rd - 28th), Kansai University Osaka (September 29th - October 4th) and at Research Institute for Mathematical Sciences Kyoto October (October 5th - 11th). While the whole trip was co-financed by various sources the ESF supported the whole stay at Kyoto next to parts of the overall travelling costs.

Purpose of the visit.

The purpose of the visit was to establish a connection to the Japanese Dirichlet form community and seek collaboration to various protagonists of this community. Particular focus of the project supported by the ESF was on the visit of Prof. Takashi Kumagai (RIMS Kyoto) with the topical focus of stability of spectral and probabilistic properties under the convergence of Dirichlet forms.

Description of the work carried out during the visit and main results obtained.

As various projects developed over the whole time span during my visit in Japan, the report is divided topically rather than chronologically.

As mentioned above, the purpose of the visit was twofold. Firstly, I received the chance to reach out to various Japanese mathematicians to discuss my research and seek their advice on several topics. Here, one has to highlight the discussions with Prof. Masatoshi Fukushima (Osaka), Prof. Takashi Kumagai (Kyoto) and Prof. Kazuhiro Kuwae (Kumamoto). The connections to these outstanding mathematicians will prove to be of great value to me in the view of future correspondence and collaboration. Secondly, I continued the research on some joint projects with people who I met during my visit to Japan. In the following I will elaborate on the details concerning these two aspects.

With Prof. Kumagai I discussed different notions of convergence of Dirichlet forms. Unfortunately many spectral properties are not stable under these forms of convergence unless one chooses proper rescalings which seem to depend heavily on the particular model. However, some of the known results let us hope that more general results in this direction may be obtained and I started to work on these problems.

With Prof. Masatoshi Fukushima and Prof. Kazuhiro Kuwae, I discussed the notion of reflected Dirichlet spaces for quasi-regular Dirichlet forms which may help to generalize recent results of [HKLMS] about global properties of regular Dirichlet forms to the quasi-regular setting.

Among the other projects I would like to point out two which seem promising:

One is a joint work with Matthias Keller (Hebrew University Jerusalem), who I met at the SAA conference in Okayama, which was started earlier this year. We were able to prove a version of the Feynman-Kac-Itō formula for magnetic Schrödinger operators on (possibly) locally infinite graphs allowing very general magnetic fields and potentials. Furthermore we established general results concerning the essential self-adjointness of the occurring operators.

The second project I would like to mention is one with Prof. René Schilling (TU Dresden) who I met at Osaka. We constructed intrinsic metrics for Dirichlet forms which are associated with Lévy type processes on $\mathbb{R}^d$. In most cases these Dirichlet forms have jump measures which are not absolutely continuous with respect to the Lebesgue measure. Such examples have not been discussed so far. We will continue this work and study the geometric properties of these metrics as well as their application to the spectral theory of the associated operators.

Future Collaborations and Project Publications.

The project on the Feynman-Kac-Itō formula will result in a publication and I expect the project on intrinsic metrics to result in a publication as well. In both cases the support of the ESF will be acknowledged.

[HKLMS] S. Haeseler, M. Keller, D. Lenz, J. Masamune, M. Schmidt *Global properties of Dirichlet forms in terms of Green's forumla.*, in preparation.