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The study of the inner model HOD is of great importance in set theory. A theorem of Vopenka says that every set is generic over HOD and a more recent result of Sy Friedman shows that V is a class generic extension of its HOD. On the other hand we can connect the study of HOD to inner models. In particular it follows from results of Steel that if there is no inner model for a Woodin cardinal then "measure one covering relative to HOD" holds: if κ is a measurable cardinal with normal measure U, then $\{\alpha < \kappa : \alpha^+ = (\alpha^+)^{HOD}\}$ has U-measure one.

It was shown by Dobrinen and Sy Friedman that this covering property can fails assuming the existence of a hyperstrong cardinal, and in fact it is consistent that $(\alpha^+)^{HOD} < \alpha^+$ for all regular cardinals α.

It is natural to ask if it is consistent to have $(\alpha^+)^{HOD} < \alpha^+$ for all cardinals α including singular ones.

In a joint work with James Cummings and Sy Friedman, we started thinking on this problem and we got a good progress on the problem. The main technique of the proof is supercompact Radin forcing. An important obstacle that appears in the proof is homogeneity property of our supercompact Radin forcing over a suitable inner model which has the same cardinals as the starting ground model. This inner model is not just the ordinary Radin extension and it must be defined more carefully.