Report on the visit of Jos Carrasquel to the Centre of Mathematics of the University of Minho (Portugal)

José Carrasquel has visited the Centre of Mathematics of the University of Minho during the period January 13-18. During the same period, José Calcines (University of la Laguna, Spain) was also a visitor of the Centre of Mathematics.

The work realized during this week has been focussed on the topological complexity or, more generally, on the sectional category. The topological complexity of a space X, $TC(X)$, is indeed the sectional category of the diagonal map $\Delta_X : X \to X \times X$. After some discussions on these notions, José Calcines, José Carrasquel and Lucile Vandembroucq have initiated a work whose starting point is the Iwase-Sakai conjecture. This conjecture asserts that the topological complexity of a space coincides with its monoidal topological complexity, $TC(X) = TC^M(X)$ (see [3]). As recently proved by A. Dranisnikov ([2]) this is true under some restriction on the dimension and the connectivity but nothing is known in general.

A weaker conjecture has been proposed by Doeraene and El Haouari ([1]): they introduced an upper bound of the topological complexity of a space X, $relcat(\Delta_X)$, which satisfies $TC(X) \leq relcat(\Delta_X) \leq TC^M(X)$ and they conjectured that $TC(X) = relcat(\Delta_X)$. Actually Doeraene and El Haouari defined, for any map f, an upperbound $relcat(f)$ of the sectional category $secat(f)$ and their (more general) conjecture asserts that, if f admits a homotopy retraction, then $secat(f) = relcat(f)$. The invariant $relcat(f)$ is defined from the characterisation by means of joins of the sectional category by requiring some extra condition on the section.

José Calcines, José Carrasquel and Lucile Vandembroucq have established weak versions of Doeraene-El Haouari's conjecture which can be seen as "linear" versions of this conjecture. For instance, if one translates the characterization/definition of $secat/relcat$ at the level of homology one gets lower bounds of these invariants, say $Hsecat$ and $Hrelcat$, and one of the linear versions of Doeraene-El Haouari's conjecture which have been established is as follows: If f admits a homotopy retraction, then $Hsecat(f) = Hrelcat(f)$. As a continuation of this work, it will be studied in which extent the results which have been obtained can be improved -in particular, in the context of the rational homotopy theory- and an article containing these results will be prepared.

During his visit, José Carrasquel also gave a seminar with the title *Rational higher topological complexity.*

References

[1] J.P. Doeraene, M. El Haouari, *When does secat equal relcat?*, to appear in Bull. Belg. Math. Soc.

[2] A. Dranishnikov, *On topological complexity and LS-category*, arXiv:1207.7309.

[3] N. Iwase, M. Sakai, *Erratum to "Toplogical Complexity is a fibrewise LS-category"*, Topology Appl. 159 (2012), 2810-2813.