# SCIENTIFIC REPORT ABOUT THE SHORT VISIT SUPPORTED BY THE ESF PROGRAM "RGLIS"

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| **Grantee:** | **Balázs Ráth** |
| **Hosts:** | **Prof. Alain-Sol Sznitman and** |
| | **Prof. Vladas Sidoravicius** |
| **Title of research project:** | **Random Interlacements / 1-dim DLA** |
| **Host institution:** | **ETH Zürich** |
| **Dates:** | **16-20 December 2012** |

## 1 Purpose of the visit

The main goal of my visit to Zürich was to continue ongoing research with Prof. Sidoravicius on the phase transition of a certain one-dimensional diffusion-limited aggregation (DLA) model defined in [4]. The model describes the growth of an aggregate interacting with a Poisson cloud of particles that perform independent one-dimensional simple random walks. One is interested in the asymptotic growth rate of the size of the aggregate $R(T)$ when $1 \ll T$, and in particular how this growth rate depends on the initial density $\mu$ of the particles. It is conjectured that the model undergoes phase transition as $\mu$ varies:

$$R(T) \asymp T^{\alpha}, \qquad \text{where} \qquad \alpha = \begin{cases} 1/2 & \text{if } \mu < 1, \\ 2/3 & \text{if } \mu = 1, \\ 1 & \text{if } \mu > 1. \end{cases} \tag{1}$$

We already have a heuristic proof of the above conjecture, which relies on the description of the hydrodynamics of the cloud of particles close to the tip of the aggregate. The goal of my visit was to see how close this argument is to being rigorous.

The secondary goal of my visit was to discuss recent progress and future plans about the geometry of the vacant set of random interlacements [5] with Prof. Sznitman, who was the postdoctoral supervisor in 2010-2012 of Artem Sapozhnikov, Alex Drewitz and myself, the authors of the recent paper [2].

## 2 Description of the work carried out during the visit

With Prof. Sidoravicius we discussed the supercritical phase of the one-dimensional DLA model, i.e., the case when the initial density of particles $\mu$ is greater than 1. It is proved in [4] that $R(T) = \mathcal{O}(T)$ as $T \to \infty$, but there is no rigorous proof yet for the corresponding lower bound, even for big values of $\mu$. Our heuristic argument predicts $\lim_{T\to\infty} \frac{R(T)}{T} = a(\mu)$ with $a(1+\varepsilon) \asymp \varepsilon$. The argument involves an approximation of the time evolution of the density profile of the particles by the solution of the heat equation with a certain "moving" Dirichlet boundary condition. During my visit to Zürich, we have found that that this heuristic approximation can indeed be made rigorous, as I will further discuss in Section 3 of this report.

With Prof. Sznitman we discussed the possible implications of the recent paper [2] on the geometry of the vacant set of random interlacements and the excursion set of the Gaussian free field. I also discussed with him our ongoing joint research with Omer Angel and Qingsan Zhu at the University of British Columbia, which is closely related to random interlacements.

## 3 Description of the main results (1-dim DLA model)

We have discovered that one of the central methods of our heuristic argument can in fact be made rigorous: after conditioning on the natural filtration generated by the process $R(T)$, the time evolution of the density profile of the particles is indeed governed by a certain controlled variant of the discrete heat equation. We have also found that the particles also possess a nice conditional independence structure and that the conditional distribution of particles is Poisson. These properties also allow us to give an intrinsic description of the stochastic process $R(T)$ which will become useful in the proof of the conjecture (1). The next step we need to take in this direction is to understand the quantitative properties of the solution of the controlled heat equation that arises from our model.

## 4 Projected publications (1-dim DLA model)

Based on the partial results already obtained during this visit (and also earlier heuristic results) we expect that our research will lead to a joint publication in 2013. The proof of the conjecture (1) may be covered in more than one paper, given the fact that the subcritical case $\mu < 1$ is more tractable than the supercritical case $\mu > 1$, and of course the most delicate case is the critical one, corresponding to $\mu = 1$.

## 5 Other comments

Besides the research and collaboration described above I also had the opportunity to meet and have fruitful discussions with multiple mathematicians who also happened to be in Zürich during my visit. In particular,

1. I have met with Jiří Černý (Universität Wien) and discussed with him our recent results [2] about chemical distances in percolation models with long-range correlations, and compared the methods and results to those of his joint paper [1] with Serguei Popov about chemical distances in the interlacement graph. This discussion may lead to a visit to Vienna in 2013.

2. I have also met with my collaborator James Martin (Oxford) and discussed with him our current research project involving the scaling limit of the coagulation-fragmentation process of big components arising from the the so-called mean field frozen percolation model defined in [3].

## References


[1] J. Černý and S. Popov (2012) On the internal distance in the interlacement set. *Electron. J. Probab.* **17**, no. 29, 1–25.

[2] A. Drewitz, B. Ráth, A. Sapozhnikov. On chemical distances and shape theorems in percolation models with long-range correlations. *(submitted)*, arXiv:1212.2885, 2012.

[3] B. Ráth. Mean field frozen percolation *J. Stat. Phys.* **137(3)**: 459-499, 2009.

[4] H. Kesten, V. Sidoravicius. A problem in one-dimensional diffusion-limited aggregation (DLA) and positive recurrence of Markov chains. *Ann. Probab.*, **36(5)**: 1838-1879, 2008.

[5] A.-S. Sznitman. Vacant set of random interlacements and percolation. *Ann. Math.* **171**: 2039-2087, 2010.