Short Visit Grant

Scientific Report

Proposal TITLE: (Thermodynamic) Bethe Ansatz and related methodologies in gauge/string correspondences: scattering amplitudes, cusp dimension and quark/ anti-quark potential.

Application reference number: 6717

1)Purpose of the visit.

Upon an invitation by the Theory Group in Budapest, I have participated in and delivered a plenary talk at the Workshop/ Conference “Finite-size technology in low dimensional quantum systems (VII)” and “Integrability in Low Dimensional Quantum Systems” in Buadapest/Tihany (Hungarian Academy of Sciences). In this

context I have enjoyed the opportunity to exchange ideas and information with several scholars on the topic of classical/ quantum integrability, especially regarding its use and application in the gauge/string correspondences.

2) Description of the work carried out during the visit.

Prof. Janos Balog, who represents the host and inviting Institution, was interested in deepening the topic of my talk on the scattering of ‘new’ kind of particles (representing some parton fields of the gauge theory) above the Gubser-Klebanov-Polyakov string vacuum solution, or, better, its quantum Bethe Ansatz representation. Although these excitations represent solution of the closed string theory, they are supposed to share many liaisons with the classical

string solution for (ending on) a null polygonal Wilson loop (gluon scattering amplitudes at strong coupling), and more in general with the null polygonal Wilson loop at al values of the coupling. The former are also represented, at least mathematically, by a relativistic Thermodynamic Bethe Ansatz (TBA), whose cluster (infra-red or large size) expansion should reveal details of the scattering at strong coupling. Besides, the latter is largely more mysterious and unknown, without enjoying any Thermodynamic Bethe Ansatz (TBA) form so far. On the contrary, it can be somehow interpreted as a sort of 2D Form Factor series representing the OPE expansion of the Wilson loop. And this series can be, both at strong and at any coupling, constructed via the 2D scattering amplitudes I have presented in my talk and discussions.

In this context we have been having

fruitful discussions with prof. Yuji Satoh , —who was visiting the Budapest Group as well —, about his recent paper on the hexagon polygon, the corresponding (lattice) perturbed 2D CFT and its Thermodynamic Bethe Ansatz (arXiv: 1406.5904 [hep-th]).

Moreover, Satoh explained to me very clearly the classical string problem of minimal area of a null polygonal Wilson loop, making evident to me that this is a sort of generalisation of a problem I dealt with in the past, i.e. the correspondence between classical (ordinary/partial) differential equations and (Quantum) Integrable Models. While Satoh gave me many suggestions from the polygonal Wilson loop side, Claire Dunning updated me on the independent and more recent developments of the ODE/IM correspondence with particular regard to the specific topic which has seen my own original contribution in the past, that of the

excited states in CFTs.

Despite the relevance of this last interpretation of the TBA for null polygonal Wilson loops at strong coupling, its interpretation as a genuine TBA emerging from scattering still eludes our efforts and hide us the connection with the scattering amplitudes of GKP excitations as we now know them.

3) Description of the main results obtained.

I have realised that the spectrum of particles at strong coupling as it emerges from the TBA coincides with that of string theory and could be seriously interpreted by expanding the TBA at large size. This expansion — about which Balog is a renewed expert who published on this specific topic and helped me in understanding it — should be compared

with the Form Factor one as expressed through our scattering matrix amplitudes. In this context a new particle which emerges only at strong coupling is a heavy boson of mass 2 represented by the Dynkin diagram central nodes; this particle seems to be generated by spurious bound states of the fermions whose scattering matrices have degenerate poles on the real axis (at infinite coupling only). All these considerations deserves a serious treatment in future publications.

Satoh's paper could be improved and generalised by using numerically some expressions found by Marco Rossi and myself in two previous papers (hep-th/0211094 and hep-th/0302220) and concerning the transfer matrix of Sine-Gordon and perturbed CFTs models. Otherwise these expressions should be expanded in the regular series (small spectral parameter), while in our papers

they are developed in the other regime, the asymptotic one (large spectral parameter). As highlighted by Satoh, the first expansion is not obvious at all from our exact expressions and we could not progress much further in the little time available.

4+5) Future collaboration with host Institution and projected publications / articles resulting from the grant.

Balog and his collaborators have a formal methodology to extract the Wilson cusp (somehow the simplest (null) Wilson polygon), in case of light-like segments, from the boundary TBA; so far they were able to expand the cusp anomalous dimension only up to two loops (arXiv: 1312.4258 [hep-th]). It is our intent to make more concrete this method with the

aim at deriving Beisert-Eden-Staudacher-like equations, also for the large spin correction, the so-called virtual scaling function, whose linear integral equation was derived by my collaborators and myself in arXiv:0802.0027 [hep-th].