European Science Foundation
Research Networking Programmes
Short Visit Grant Scientific Report

Martino Lupini

October 13th, 2013

Proposal title

Borel complexity of categories

Application reference number

5541

Purpose of the visit

The goal of my visit was to attend the workshops on Forcing and Large Cardinals and on Descriptive Set Theory held at the Erwin Schrödinger International Institute for Mathematical Physics (ESI) in Vienna. Moreover during my visit I aimed at collaborating with my hosts on a research project on Borel complexity of categories.

Description of the work carried out during the visit

The workshops at the ESI have gathered together the leader experts in the fields of Large Cardinals and Descriptive Set Theory. They gave talks of extremely high quality, presenting their most recent results. Attending the workshops has allowed me to get familiar with the latest developments in Set Theory. Moreover during my stay in Vienna I have obtained progress on the problem of determining whether the hyperfinite separable II_1 factor has a generic masa.

Description of the main results obtained

Finite von Neumann algebras can be regarded as the noncommutative analogue of probability spaces. The commutative finite von Neumann algebras are exactly those ones of the form $L^{\infty}(\mu)$ for some probability measure μ. Every finite von Neumann algebra can be decomposed as a direct integral of finite factors, which are finite von Neumann algebras with trivial center. Therefore factors play a key role in the classification of finite von Neumann algebras. The infinite dimensional finite factors are called II_1 factors. Arguably the most important among II_1 factors is the so called hyperfinite separable II_1 factor, usually denoted by $\mathcal{R}$, obtained as a direct limit of a sequence of matrix algebras. A cornerstone result of Murray and von Neumann dating back to the 1930s asserts that any II_1 factors contains a copy of $\mathcal{R}$, see [1, Theorem XIII]. Moreover the so called Connes' Embedding Problem asks whether every separable II_1 factors can be embedded in an ultrapower of $\mathcal{R}$; Formulated almost 40 years ago by Alain Connes in [2], the Connes Embedding Problem is currently the most important open problem in the theory of II_1 factors. These and many other observations and results contributed to the interest of researchers in understanding $\mathcal{R}$ and its substructures. Masas, i.e. the maximal elements in the poset of abelian selfadjoint subalgebras of M ordered by inclusion, are among the most important substructures of a given separable II_1 factor M. Two masas A and B of M are usually identified when they sit in the same way inside M, i.e. there is an automorphism of M sending A onto B. This defines a natural equivalence relation called conjugacy on the Polish space of masas of M. David Kerr and Stuart White have recently obtained as an application of the theory of turbulence developed by Hjorth in [3] the following result: If the conjugacy classes of masas in a separable II_1 factor M are meager, then the masas of M are not classifiable by countable structures up to conjugacy. Moreover they observed that whenever M has a countable outer automorphism group, then the conjugacy classes of masas of M are meager; They was not able to show that the same fact holds for an arbitrary separable II_1 factor M. In particular this problem remained unsolved for the hyperfinite separable II_1 factor, since it is a well known fact that the outer automorphism group of $\mathcal{R}$ is uncountable. It was then believed that the conjugacy classes of masas in the hyperfinite separable II_1 factor might not be meager, and in fact there might be a generic masa. Asger Törquist suggested that one could try to show this by means of Fraïssé theory for metric structures as developed by Ben Yaacov in [4] and with a more category-theoretic approach by Kubiś in [5]. During my stay in Vienna I have explored this new promising approach, building a generic masa of $\mathcal{R}$ as a metric Fraïssé limit of a suitable class of structures.

Future collaboration with host institution (if applicable)

I might attend the workshop "Geometry of computation in groups" to be held at the ESI from March 31st to April 4th, 2013.

Projected publications / articles resulting or to result from the grant

"The hyperfinite separable II_1 factor has a generic masa"

References

[1] F. J. Murray and J. von Neumann. On rings of operators. IV. Ann. of Math. (2), 44:716-808, 1943.

[2] A. Connes. Classification of injective factors. Cases II_1, II_∞, III_λ, $\lambda \neq 1$. Ann. of Math. (2), 104(1):73-115, 1976.

[3] G. Hjorth, Classification and orbit equivalence relations, Mathematical Surveys and Monographs, vol. 75, American Mathematical Society, Providence, RI, 2000.

[4] Ben Yaacov, Fraïssé limits of metric structures, preprint, available at `http://math.univ-lyon1.fr/\char126\relaxbegnac/articles/Fraisse.pdf`

[5] W. Kubiś, Metric-enriched categories and approximate Fraïssé limits, preprint, available at arXiv:1210.6506