SHORT VISIT GRANT - FINAL REPORT

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Proposal Title: Topology and Geometry of Groups.

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1. Purpose of the visit.

The purpose of the visit was to work in close collaboration with V. Poénaru (Emeritus Professor at the University of Paris-Sud 11), on some problems related to the asymptotic topology of groups. The central themes of our research were in particular two tameness conditions for manifolds and discrete (finitely presented) groups: the QSF property (*quasi-simple filtration*) and the notion of *easy-representability.*

2. Description of the research done.

Poénaru's first results on the topology at infinity of universal covering spaces of closed 3-manifolds worked 'approximating' them by compact and simply-connected sub-manifolds (see [8, 9, 13] and [3]); the idea of this condition was then refined and adapted for arbitrary finitely presented groups by Brick and Mihalik, under the name *quasi-simply filtration* or QSF (which basically amounts to finding an exhaustion 'approximable' by finite, simply-connected complexes) [1, 15].

On the other hand, one of Poénaru's methods for proving the QSF for several low-dimensional objects was to 'represent' them in a special "*easy*" way, such as homotopy 3-spheres, or universal covering spaces of 3-manifolds [10, 14], and, recently, discrete groups too [11], where such an *(inverse)-representation* for a finitely presented group is, roughly speaking, a quasi-surjective map from some QSF 2-complex to a certain singular 3-manifold canonically associated to the group, satisfying several topological properties, and for which the set of double points is closed (see also [7, 12]).

This condition turns out to be very relevant for finitely presented discrete groups, and in fact, in collaboration with Poénaru, we actually proved that *easy groups*, i.e. groups admitting such an easy-representation, are QSF [5].

Notice that all these topological properties (that are different for general spaces) may be compared with each others in the realm of finitely presented groups, since it turns out that in this setting there exists the notion of '*(almost)-equivalence*' for topological properties of groups (i.e two such conditions are almost equivalent whenever they "define the same class of groups", see [2]).

Now, our main (conjectural) idea is that the QSF property should be equivalent, for discrete finitely presented groups (see [2]), to the notion of easy-representability.

This was actually the starting point for our investigations together, since Poénaru is the expert of the inverse easy-representability condition of manifolds and groups: we needed to make the point of our research and to elucidate some details and ideas.

3. Achievements and resulting articles.

In order to prove the easy-QSF equivalence, independently of the very general result of Poénaru [11], what we need is the arrow 'QSF implies easy' (the other one being [5]). Now, the results of [2] on the equivalence of the QSF property with the Tucker property, together with those of [4] on group combings, get that one needs to show that groups admitting a tame 1-combing (as defined in [4]) are easy.

To start, during the short visit to the University of Paris-Sud 11 (Orsay), we analyzed the easiest combings, as those of [9]. In order to obtain the expected result, we used and followed the techniques of [5, 9]. Precisely, the main result we obtained is the following:

Theorem 1 (Otera-Poénaru, [6], 2013)**.** *Groups admitting a Lipschitz 0-combing (as defined in* [9]*) are easy.*

This Theorem and its proof are contained in the manuscript [6] "*Tame combings and easy groups*" by D.E. Otera and V. Poénaru (the ITGP program grant and the ESF are acknowledged in it). This preprint is now submitted for a possible publication.

4. Future collaboration.

Once this result achieved, we actually plan to continue our research in collaboration with V. Poénaru and with the University of Paris-Sud 11. More precisely, we will try to extend this result for the far more general *tame 1-combable groups* (as defined in [4]), and in particular, we plan to show that Tucker groups are easy. This should be the main point for proving that QSF groups are easy, and will be the object of a future collaboration with V. Poénaru at the University of Paris-Sud 11 during the next year.

References

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